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MATHEMATICAL REVIEWS

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Mathematical Reviews

Vol. 22, No. 5B May, 1961 Reviews 4085–4613

PROBABILITY

See also A3761, A3762, A4019, 4103, 4280, 4578, 4582.

4085:

da Veiga de Oliveira, Fernando Alves. A first lesson in the calculus of probabilities. Ciência. Lisboa No. 15/16 (1958/59), 53–66. (Portuguese)

Expository.

4086:

Conolly, B. W. The busy period in relation to the single-server queueing system with general independent arrivals and Erlangian service-time. J. Roy. Statist. Soc. Ser. B **22** (1960), 89–96.

This paper is concerned with busy period distributions for a single-server queue, with a general independent input, and Erlangian service time distribution. It is used here to indicate the service time of a group of customers. The principal interest of the paper is to compute the joint probability and joint density that during a busy period (the period in which the server is occupied without a break), exactly m groups each of size k, receive service, and the whole process occupies time t. The marginal distributions of m and t are also discussed and the ideas are applied to known special cases. The procedure used is a modification of the argument used by the author in a previous work [Biometrika **46** (1959), no. 1/2, 246–251; MR **21** #1650]. Thus he considers arrival instants and specifies sufficient variables to ensure that the process is Markovian. Here his auxiliary functions include added information on the member of a group and on the group which is undergoing service at the time considered.

T. L. Saaty (Silver Spring, Md.)

4087:

Miller, Rupert G., Jr. A contribution to the theory of bulk queues. J. Roy. Statist. Soc. Ser. B **21** (1959), 320–337.

The author discusses two models for bulk queues. A 'bulk queueing problem' describes a process whereby groups of items arrive at a service station with both the size of the arrival group and the interval between arrivals of different groups being randomly distributed. Likewise, in the general model, the service station will handle more than one item at a time, but both the size of the service groups and the length of the servicing intervals are randomly distributed. The two models discussed differ in the following way: In one model if a group arrives where there are no items waiting and the number of items being served is less than the service group quota, then as many items as are needed to fill the quota are taken immediately into service from the arrival group without affecting the length of the service period. In the second model, all arrivals during a service period must wait until the end of the period whether the service quota is filled or not.

For the first model, the author determines conditions under which the queue will vanish with probability one in a finite time and when the period between empty queues will have a finite expected value. As could be expected it depends on the traffic intensity, ρ, being <1. However, there are no major restrictions, other than independence of variables, on the various distributions. For the second model similar conditions are determined if the arrival distribution is exponential.

For the case of exponential arrival for the second model, the author also analyzes the distribution of the "imbedded Markov chains" which consider the states only at the ends of service periods. For this model when all service groups are single items, he examines the waiting time and busy period distributions. For the first model with exponential arrivals, the author analyzes the distribution and waiting line properties of the imbedded chain considering the states only at the times of arrival of new groups.

H. M. Gurk (Princeton, N.J.)

STATISTICS

See also 4131, 4132.

4088:

Stange, K. Spannweitenpläne für messende Prüfung im doppelten Wahrscheinlichkeitsnetz. Metrika **3** (1960), 151–165. (English summary)

Author's summary: "In using a Range-Plan the variability of the characteristic x is estimated by the mean range of l subsamples of size m. The parameters of such a plan (sample size n and acceptability constant K) can be determined for any OC-curve by means of the "Double-Probability-Paper" with no calculations at all. This graph paper transforms the OC as given by the two points $(p_0; 1-\alpha)$ and $(p_1; \beta)$ into a straight line provided that $n \gtrapprox 15$ and $m \gtrapprox 5$."

4089:

Roy, A. R.; Mohanty, S. G. Distribution of χ^2-analogue for normal population with class intervals defined in terms of sample median. J. Indian Soc. Agric. Statist. **10** (1958), 90–98, xix. (Hindi summary)

A. R. Roy [*On statistics with variable intervals*, Stanford University Tech. Rept. No. 1-Nonr-225 (21) (1956)] and

G. S. Watson [J. Roy. Statist. Soc. Ser. B **20** (1958), 44–72; MR **20** #6745] independently derived the asymptotic distribution of the chi-square goodness of fit statistic when the intervals and estimated probabilities were permitted to depend on the maximum likelihood estimate of an unknown scale or location parameter. This result is extended by means of an example involving a normal distribution with unknown variance where the (three) intervals and estimated probabilities depend on the sample median. The asymptotic distribution is that of $\lambda_1 x_1^2 + \lambda_2 x_2^2$, where x_1 and x_2 are independent standardized normal variates, and $\lambda_1 \geq 1$ and $0 \leq \lambda_2 \leq 1$. This is compared with the case where maximum likelihood is applied and consequently $\lambda_1 = 1$ and $0 \leq \lambda_2 \leq 1$.

H. Chernoff (Stanford, Calif.)

4090:

Kabe, D. G. Some applications of integral transforms to distribution problems in statistics. J. Karnatak Univ. **3** (1958), no. 1, 106–112.

Expository note dealing with the application of Fourier and Mellin transforms to certain distribution problems.

Benjamin Epstein (Palo Alto, Calif.)

4091:

Fabian, Václav. Stochastic approximation methods. Czechoslovak Math. J. **10** (85) (1960), 123–159. (Russian summary)

From the author's summary: "In the present paper we propose two modifications of the known procedures and study their convergence with probability one. The recurrence relation for the Robbins-Monro procedure can be rewritten in the form

$$X_{n+1} = X_n + a_n |Y_n| \operatorname{sign} Y_n,$$

where Y_n is an estimate of $-R(X_n)$. We determine the length of the nth move to be a_n instead of $a_n|Y_n|$, weakening conditions previously imposed on R. The situation in the case of the Kiefer-Wolfowitz method is analogous.

"The second modification is motivated by the fact that in the search for a minimum of a function by the method of Blum we need at least $q+1$ observations for determining the direction at each step. We propose to determine the length α_n in the following way: If X_n and Y_n are observed, take observations V_j (independent of X_n, Y_n) of $R(X_n + ja Y_n)$ for $j = 1, 2, \cdots$ until $V_1 > V_2 > \cdots > V_{j-1}$ and put $\alpha_n = ja$ if $V_1 > V_2 > \cdots > V_{j-1} > V_j \leq V_{j+1}$."

J. Kiefer (Ithaca, N.Y.)

4092:

Lehmann, E. L. Optimum invariant tests. Ann. Math. Statist. **30** (1959), 881–884.

Suppose that a hypothesis testing problem is invariant under a group G for which the usual invariance theory holds, and that an order relation $\underset{\sim}{\prec}$ is defined on a class T of tests ϕ invariant under G such that $\phi \underset{\sim}{\prec} \phi'$ whenever (i) ϕ' is uniformly more powerful than ϕ, or (ii) ϕ' is a probabilistic average of tests ϕ_γ for which $\phi \underset{\sim}{\prec} \phi_\gamma$, or (iii) the power function of ϕ' is the limit of those of the sequence $\{\phi_n\}$, where $\phi \underset{\sim}{\prec} \phi_n$, and such that $\phi \underset{\sim}{\prec} \phi'$ implies $\phi g \underset{\sim}{\prec} \phi' g$ for all g in G. ϕ_0 is said to be optimum $(\underset{\sim}{\prec}, T)$ if $\phi \underset{\sim}{\prec} \phi_0$ for all ϕ in T. The author proves that there then exists a G-invariant test which is optimum $(\underset{\sim}{\prec}, T)$. One consequence is the validity of all known optimum properties of the analysis of variance test for the general univariate linear hypothesis, e.g., Wald's theorem [same Ann. **13** (1942), 434–439; MR **5**, 129] or the reviewer's result [ibid. **29** (1958), 675–699; MR **20** #4910] that the test is of type E.

J. Kiefer (Ithaca, N.Y.)

4093:

Williams, E. J. The comparison of regression variables. J. Roy. Statist. Soc. Ser. B **21** (1959), 396–399.

Several predictor variates are available for predicting the value of another variate. Hotelling [Ann. Math. Statist. **11** (1940), 271–283; MR **2**, 111] developed a test to decide whether the predictors differ in their efficiency of prediction. Hotelling's test was formulated in terms of correlations. The present paper reformulates the test in terms of regression analysis. Calculations are probably simpler using this new formulation.

S. W. Nash (Ames, Iowa)

4094:

Walsh, John E. Nonparametric tests for median by interpolation from sign tests. Ann. Inst. Statist. Math. **11** (1960), 183–188.

Exact and approximate generalizations of the sign test are proposed. Assume $x_1, \cdots, x_n$ are the ordered observations in a sample from a population with a symmetric density. If the test based on $x_i > 0$ has an alpha which is too small and the test based on $x_{i+1} > 0$ has an alpha which is too large, then the following kinds of tests are considered. Reject the null hypothesis if $\frac{1}{2}(x_i + x_{i+1}) > 0$; this will be a similar test and may have a more desirable alpha. Generally, for $0 < A < 1$, the test based on $Ax_i + (1-A)x_{i+1} > 0$ will not be similar but it should have an alpha approximated by $AP(X_i > 0) + (1-A)P(X_{i+1} > 0)$.

I. R. Savage (Cambridge, Mass.)

4095:

Tukey, John W. Some examples with fiducial relevance. Ann. Math. Statist. **28** (1957), 687–695.

Im Zusammenhang mit der Fiduzialtheorie ist die Frage wichtig, ob unter den dort üblichen Annahmen (Suffizienz, eindeutige Umkehrbarkeit, Kontinuität) die induzierte Verteilung eindeutig bestimmt ist. Es sind schon Beispiele dafür bekannt [J. G. Mauldon, J. Roy. Statist. Soc. Ser. B **17** (1955), 78–85; MR **17**, 380], daß dies nicht der Fall ist. Verf. konstruiert ein einfacheres Beispiel, das auf Drehung zweidimensionaler Normalverteilungen beruht, und zeigt, daß die klassische Verteilung beim Behrens-Fisher-Problem nicht die einzig mögliche ist. Ein bisher unveröffentlichtes Beispiel für Nichteindeutigkeit von L. J. Savage (1952) und ein Literaturverzeichnis von 33 Titeln über Fiduzialtheorie bilden den Abschluß. Einige Druckfehler.

O. Ludwig (Zbl **82**, 136)

4096:

Weiss, Lionel. A test to fit based on the largest sample spacing. J. Soc. Indust. Appl. Math. **8** (1960), 295–299.

Let V be the largest of the $n+1$ sub-intervals of $[0, 1]$ induced by n independent random variables each with the distribution function $F(x)$ carried on $[0, 1]$. The author discusses a test that $F(x) = x$, $0 \leq x \leq 1$, based on its rejection if V is too large. The test is shown to be admissable and consistent.

D. A. Darling (Ann Arbor, Mich.)

4097:

Pfanzagl, J. Über lokal optimale Rang-Tests. Metrika **3** (1960), 143–150. (English summary)

The absolute value of one-sided, locally most powerful, rank order test statistics is shown to have an optimal property for two-sided alternatives. This property was not considered by Lehmann [Ann. Math. Statist. **24** (1953), 23–43; MR **14**, 888]. Particular attention is devoted to the Wilcoxon-Mann-Whitney test.

I. R. Savage (Cambridge, Mass.)

4098:

Steel, Robert G. D. A rank sum test for comparing all pairs of treatments. Technometrics **2** (1960), 197–207.

Author's summary: "A multiple comparison rank sum test, for the simultaneous comparison of all pairs of treatments in a one-way classification with equal numbers of observations, is presented. An example is worked and tables of critical values are given. Computation of probabilities for the general case of unequal numbers of observations is considered and means, variances, and covariances are given for this case." *J. Kiefer* (Ithaca, N.Y.)

4099:

Bechhofer, Robert E. A multiplicative model for analyzing variances which are affected by several factors. J. Amer. Statist. Assoc. **55** (1960), 245–264.

From the author's summary: "In this paper we propose a new way of looking at multifactor experiments which are conducted to study the effect of changes in the levels of the factors on the variance of a chance variable. A mathematical model is proposed which assumes that the effect of changing a factor from "level" 1 to "level" 2 is to multiply the original variance of the chance variable by a positive constant. Factorial experiments, experimental designs, and the concept of an approximating multiplicative response surface for variances are described. It is shown how the use of the log transformation of the sample variances, when the underlying variables are normally distributed, makes it possible to employ standard procedures for analyzing the data and thereby obtain excellent approximate results. Consistent point estimates of the parameters of interest are proposed. It is shown how a large class of hypothesis-testing problems can be viewed as a special case of what is termed the univariate general multiplicative hypothesis for variances —an analogue of the univariate general linear hypothesis for means. The multiplicative model for variances is generalized to the multivariate case."

J. Kiefer (Ithaca, N.Y.)

4100:

Hill, Hubert M. Experimental designs to adjust for time trends. Technometrics **2** (1960), 67–82.

The author considers designs of the type constructed by Box [Biometrika **39** (1952), 49–57, and elsewhere] and Cox [Biometrika **38** (1951), 312–323; MR **13**, 669; and elsewhere] for the estimation of regression coefficients in the presence of a time trend, and also combinations of these designs. There is no discussion of optimality (the class of available designs in fact not being delimited by the author in the Boxian case), and attention is focused on designs for which the estimates of the regression coefficients of interest are orthogonal to those of time trend, although (as the reviewer has pointed out) the restriction to such designs results in a great loss of efficiency in such settings. *J. Kiefer* (Ithaca, N.Y.)

4101:

Kiefer, J. Optimum experimental designs. J. Roy. Statist. Soc. Ser. B **21** (1959), 272–319.

A partly expository paper in which the author treats the problems of experimental design from the point of view of modern statistical decision theory. After some introductory remarks, section 2 concerns itself with the non-optimality of certain classical designs and their misuse in certain experimental situations. Section 3 treats admissible procedures and complete classes, and in section 4 designs are found which satisfy particular optimality criteria. The paper contains a number of illustrative examples.

The paper is followed by a discussion in which the participants contribute a number of irrelevant remarks. These are effectively disposed of in a reply by the author.

J. R. Blum (Albuquerque, N.M.)

4102:

Jackson, J. Edward. Bibliography on sequential analysis. J. Amer. Statist. Assoc. **55** (1960), 561–580.

Author's summary: "This article contains 374 references dealing with the subject of sequential analysis. These references are listed alphabetically by author and also broken down into fourteen subject categories."

4103:

Vogel, Walter. Ein Irrfahrten-Problem und seine Anwendung auf die Theorie der sequentiellen Versuchs-Pläne. Arch. Math. **11** (1960), 310–320.

One of two coins is flipped at each instant $k=1, 2, \cdots$. If the kth flip is made with coin number 1 [resp., 2], let $Z_k=1$ or -1 [resp., -1 or 1] according to whether the kth flip is a head or a tail. The conditional probability that the kth flip is a head, given that it is on coin i and given any past history, is p_i. The $(k+1)$st flip is made with coin 1 [resp., 2] if $V_k=\sum_1^k Z_i>0$ [resp., <0], while each coin is chosen with probability $\frac{1}{2}$ if $V_k=0$. This rule has been suggested as one which has good properties compared with other sequential designs for maximizing the number of heads in n tosses, with an appropriate loss function, and is optimum if it is known a priori that $p_1<p_2$ and $p_2<p_1$ are equally likely and $p_1+p_2=1$. [See Bradt, Johnson, and Karlin, Ann. Math. Statist. **27** (1956), 1060–1074; MR **19**, 333; Vogel, ibid. **31** (1960), 430–443.] By studying the random walk $\{V_k\}$ described above, the author determines the limiting behavior of $\sum_1^n\{P(V_k<0)+P(V_k=0)/2\}$, and thus of the risk function of the suggested procedure, as $n\to\infty$.

J. Kiefer (Ithaca, N.Y.)

4104:

Kurlat, S.; Springer, M. D. Sequential analysis of the reliability of an antitank-mine simulator system. Operations Res. **8** (1960), 473–486.

4105:

Drenick, R. F. Mathematical aspects of the reliability problem. J. Soc. Indust. Appl. Math. **8** (1960), 125–149.

In this paper several standard problems in reliability are

brought together by defining reliability in terms of an expected value of random variables associated with a renewal process. Within this same framework new problems involving preventive maintenance are formulated. That aspect of reliability concerned with the prediction of the behavior of systems in environments differing (more severe or complex) from the experimental environment is not mentioned. *I. R. Savage* (Cambridge, Mass.)

4106:

Birnbaum, Z. W.; Saunders, S. C. A statistical model for life-length of materials. J. Amer. Statist. Assoc. **53** (1958), 151–160.

A statistical model for life-lengths of structures under dynamic loading is derived. The model comprises exponential, normal and Weibull distributions as special cases and leads to a wide family of skewed distributions. The model makes it possible to express the probability distribution of life-lengths in terms of the load given as a function of time and of deterioration occurring in time independently of loading. The special case of a constant load or of periodic loading with constant amplitude leads to gamma distributions for life-lengths. The case of load functions which vary periodically or aperiodically can be extended to the case of random loading. The application of resulting probability distributions in the three series of carefully controlled experiments yields very good agreement with empirical data. *J. Janko* (Prague)

4107:

Epstein, Benjamin. Tests for the validity of the assumption that the underlying distribution of life is exponential. I. Technometrics **2** (1960), 83–101.

The author considers data consisting of the times of successive replacements in a renewal process, or a set of independent life-times following the same distribution, and describes twelve tests that such a distribution is negative-exponential. He discusses the power of such tests against alternative hypotheses such as that the distribution differs from the negative exponential or that there exists a trend. *P. A. P. Moran* (Oxford)

4108:

Epstein, Benjamin. Tests for the validity of the assumption that the underlying distribution of life is exponential. II. Technometrics **2** (1960), 167–183.

In I [preceding review], the author presented the theory of tests for exponential life distributions. He discussed graphical procedures, the χ^2 test of goodness of fit, tests based on the conditional distribution of total lives, tests for abnormally early or abnormally late failures, tests for whether the mean life fluctuates, tests for abnormally long periods without failures, and the Kolmogorov-Smirnov test. In II he illustrates these tests with instructive examples and comments.

L. A. Aroian (Los Angeles, Calif.)

4109:

Fava, Lindo. The variance of the regression estimate in random sampling without replacement from a finite population. Bol. Soc. Mat. São Paulo **10** (1955), 121–128 (1958). (Portuguese. English summary)

In double sampling with an auxiliary variate [W. G. Cochran, *Sampling techniques*, Wiley, New York, 1953; MR **14**, 887] a restriction of the values of X in the infinite population to those of the preliminary sample produces an unbiased estimate of the mean Y-value in the population and an exact value of its variance.

H. L. Seal (New Haven, Conn.)

NUMERICAL METHODS

See also A3808, A3809, 4306, 4371.

4110:

Lyusternik, L. A.; Sobolev, S. L. Some problems of computational mathematics. Vestnik Akad. Nauk SSSR **30** (1960), no. 10, 23–31. (Russian)

Expository.

4111:

Palásti, Ilona; Rényi, Alfréd. Monte-Carlo methods as minimax strategies. Magyar Tud. Akad. Mat. Kutató Int. Közl. **1** (1956), 529–545 (1957). (Hungarian. Russian and English summaries)

From the authors' summary: "Monte-Carlo methods are considered in the present paper from the point of view of the theory of games. As an example, the numerical approximation of the integral $I=\int_0^1 f(x)dx$ of the continuous function $f(x)$ by the sum $S=(1/n)\sum_{k=1}^n f(x_k)$ is considered. A pure strategy of the player B consists in the choice of a function $f(x)$. It is supposed that the set Φ of admissible functions $f(x)$ consists of all continuous functions satisfying the condition $\int_0^1[f(x)-\int_0^1 f(t)dt]^2dx=s^2$, where $s>0$ is a given constant. A pure strategy of player A consists in the choice of an n-tuple of points $(x_1, x_2, \cdots, x_n)$ of the interval $(0, 1)$. The loss of A is defined by $\Delta=(S-I)^2$. A mixed strategy of A is clearly determined by a measure μ defined on the measurable subsets of the unit cube K_n of n-space, and if player A chooses the mixed strategy characterized by the measure μ his average loss is $\Delta(f,\mu)=\int_{K_n}(S-I)^2d\mu$.

"It is shown in the paper that a minimax strategy for player A is to choose the mixed strategy corresponding to the ordinary Lebesgue measure μ_0 in K_n, i.e., to play a 'Monte-Carlo' strategy. The same holds for an r-dimensional integral ($r=2, 3, \cdots$). The mean error is, independently of r, equal to $s/\sqrt{n}$. In connection with this fact, it has been frequently emphasized, that the Monte-Carlo method is advantageous only for many-dimensional integrals. In the present paper it is pointed out that the same advantages present themselves for ordinary one-dimensional integrals, provided that the function to be integrated is very strongly oscillating.

"The analogous problem of estimating the finite sum $Y=\sum_{k=1}^N y_k$ by $\eta=(N/n)\sum_{j=1}^n y_{k_j}$, where $\mathbf{k}=(k_1, k_2, \cdots, k_n)$ is some subset of the set $(1, 2, \cdots, N)$, is also considered.

"Finally account is given of some experimentation with Monte-Carlo methods."

4112:

Davis, D. H. Monte Carlo calculation of molecular flow rates through a cylindrical elbow and pipes of other shapes. J. Appl. Phys. **31** (1960), 1169–1176.

Author's summary: "A method is devised for the calculation of molecular flow rates through pipes where the mean free path for intermolecular collisions is large compared to the dimensions of the pipes. Results of the calculation are given for a straight cylindrical pipe, a cylindrical elbow, the annulus between two concentric cylinders, a straight cylindrical pipe with restricted openings, and a straight cylindrical pipe with restricted openings and a plate to block the direct beam between the openings."

4113:

Varga, Richard S. Orderings of the successive overrelaxation scheme. Pacific J. Math. **9** (1959), 925–939.

The author considers iterative methods for solving $x=Bx+g$, where x is an unknown column matrix, g is a given column matrix, B is a given square, irreducible and symmetric matrix whose elements are non-negative and whose diagonal elements vanish. The successive overrelaxation method of iteration is defined by

$$x^{(n+1)} = \omega[L_\phi x^{(n+1)} + L_\phi^T x^{(n)} + g] + (1-\omega)x^{(n)},$$

where $B=L_\phi+L_\phi^T$ and where L_ϕ is a strictly lower triangular matrix. Here ϕ denotes the ordering of the rows and columns of B. To study the convergence of the method one considers the equivalent form $x^{(n+1)}=\mathscr{L}_{\phi,\omega}x^{(n)}+f$, where $\mathscr{L}_{\phi,\omega}=(1-\omega L_\phi)^{-1}[\omega L_\phi^T+(1-\omega)I]$, $f=\omega(1-\omega L_\phi^{-1})g$. The author defines "$h$-consistent" orderings for matrices which are "cyclic of index 2" in the sense of Romanovsky [Acta. Math. **66** (1936), 147–251] or, equivalently, matrices with "Property (A)" in the sense of Young [Trans. Amer. Math. Soc. **76** (1954), 92–111; MR **15**, 562]. Orderings which are consistent in the sense of Young [loc. cit.] are h-consistent. The author shows that if $\bar{\mu}[B]<1$ then $\bar{\mu}^2[B]\leq\bar{\mu}[\mathscr{L}_{\phi,1}]$ whether or not B is cyclic of index 2, where, in general, $\bar{\mu}(A)$ denotes the spectral radius of the matrix A. (Note that $\mathscr{L}_{\phi,1}$ is the matrix corresponding to the Gauss-Seidel method.) Moreover, $\bar{\mu}^2[B]=\bar{\mu}[\mathscr{L}_{\phi,1}]$ if and only if B is cyclic of index 2 and ϕ is h-consistent. This generalizes some preliminary results on optimal orderings and proves a conjecture of Young [Thesis, Harvard, 1950]. The author also shows that the rate of convergence of the Gauss-Seidel method is asymptotically (as $\bar{\mu}[B]$ increases to 1) equal to twice the rate of convergence of the Jacobi method, whose matrix is B. This result was previously known to be true under the assumptions that B was cyclic of index 2 and that the ordering was consistent. The more general result was conjectured by Shortley and Weller [J. Appl. Phys. **9** (1938), 334–344], for the Dirichlet problem. The author gives numerical results for Laplace's equation in the rectangle both for the usual 5 point difference formula, where B is cyclic of index 2, and for the 9 point formula where it is not. The effect of the orderings on the convergence of the Gauss-Seidel is observed to be small, as predicted.

In connection with the successive overrelaxation method it has been shown by Young [loc. cit.] that if B is cyclic of index 2 and if the ordering ϕ is consistent, then the optimum relaxation factor is given by

$$\omega_b = 1+[\bar{\mu}[B]/(1+(1-\bar{\mu}^2[B])^{1/2})]^2$$

and $\bar{\mu}[\mathscr{L}_{\phi,\omega_b}]=\omega_b-1$. The author shows that in general $\mathrm{Min}_\phi[\mathrm{Min}_\omega\,\bar{\mu}(\mathscr{L}_{\phi,\omega})]\geq\omega_b-1$, and that equality cannot hold unless B is cyclic of index 2. Moreover, in this case $\bar{\mu}(\mathscr{L}_{\phi,\omega})=\omega_b-1$ if and only if $\omega=\omega_b$ and ϕ is h-consistent. The author concludes the paper by showing the applicability of the results to problems involving elliptic partial differential equations. *D. M. Young* (Austin, Tex.)

4114:

Zurmühl, Rudolf. Zur Iteration einzelner Eigenwerte von Matrizen. Z. Angew. Math. Mech. **37** (1957), 228.

Given an approximate value l of the eigenvalue λ of matrix A. Construct the iterative sequence $\lambda_i\to\lambda$, $x_i\to x$ $(Ax=\lambda x)$ by the formula $(A-\lambda I)z_{i+1}-\varepsilon_{i+1}x_i+y_0=0$, where $y_0=(A-lI)x_0, z_i=x_i-x_0$. Here all x_i $(i=0, 1, 2, \cdots)$ are normalized to have last component equal 1 (if necessary the rows and columns of A may be permuted before beginning). For convenience in computation, x_0 can be chosen also so that all components of y_0 except the last are zero.

A method very close to that described has already been published by the author [*Praktische Mathematik für Ingenieure und Physiker*, Springer, Berlin, 1953; MR **15**, 470]; the improvement made in this note consists in subjecting only one vector x_i to the iteration, rather than two as in the previous variant of the method.

M. L. Brodskiĭ (RŽMat **1958** #1597)

4115:

Causey, Robert L.; Henrici, Peter. Convergence of approximate eigenvectors in Jacobi methods. Numer. Math. **2** (1960), 67–78.

The authors study the rates of convergence of several variants on the well-known Jacobi method for finding the eigen-values and vectors of a symmetric matrix. The variants considered are three in number: the ordinary Jacobi method; a quasi-cyclic Jacobi method; and a threshold cyclic method.

H. H. Goldstine (Yorktown Heights, N.Y.)

4116:

Osborne, M. R. h^2-extrapolation in eigenvalue problems. Quart. J. Mech. Appl. Math. **13** (1960), 156–168.

The method of the deferred approach to the limit is investigated for eigenvalues and eigenfunctions of the differential equation

$$\phi''+r(x)\{\lambda-q(x)\}\phi = 0,$$

with homogeneous boundary conditions involving the function and its first derivative at $x=0$ and $x=1$. If $q(x)$ is regular, with a non-zero radius of convergence at every point in (0, 1), it is shown that central-difference methods give results, for λ and ϕ, which differ from the true results by a convergent even power series in h. The coefficients themselves satisfy differential equations, given explicitly, and are used to correct approximations without reducing the interval. There is a numerical example and a discussion of an interesting relation between some of the coefficients of the error term and the "difference correction" of some known finite-difference methods.

L. Fox (Oxford)

4117:

Saxena, R. B. On Simpson's formula of cubature. Ann. Polon. Math. **6** (1959), 289–293.

For functions $f(x, y)=\sum a_{ik}x^iy^k$ with i, k running

through a countable set of positive numbers a nine-point rectangular grid integration formula for the integral of f over a rectangle is developed. The formula makes use of the two smallest numbers p, q of the set in which the i, k vary. The weights depend on p, q but not on the coefficients a_{ik}. *H. Bückner* (Madison, Wis.)

4118:

Fehlberg, Erwin. Neue genauere Runge-Kutta-Formeln für Differentialgleichungen zweiter Ordnung. Z. Angew. Math. Mech. **40** (1960), 252–259. (English and Russian summaries)

From the author's summary: "By an m-fold differentiation and suitable transformation of a 2nd-order differential equation Runge-Kutta formulae for the transformed differential equation can be obtained which correctly represent the h^{m+5} terms inclusively in the Taylor series for the solution as well as its derivative."

R. W. Hamming (Stanford, Calif.)

4119:

York, William C.; Shu, H. Hunter; Rouleau, Wilfred T. Remarks on the numerical integration of the equations of droplet growth. J. Meteorol. **17** (1960), 456–459.

4120:

Lotkin, Mark. The calculation of heat flow in melting solids. Quart. Appl. Math. **18** (1960/61), 79–85.

In a previous paper [J. Math. Phys. **37** (1958), 178–187; MR **20** #5562] the author treated the numerical integration of the heat conduction equations for the transfer of heat in a finite slab composed of materials with variable thermal properties. The present paper extends these results to include the phenomenon of melting at a face of the slab. *J. Elliott* (New York)

4121:

Oulès, Hubert. Sur l'intégration numérique de l'équation intégrale de Volterra de seconde espèce. C. R. Acad. Sci. Paris **250** (1960), 1433–1435.

The integral in a Volterra integral equation of second kind is computed over an interval of length h with an error of $O(h^5)$ by means of a Runge-Kutta-like formula.

H. Bückner (Madison, Wis.)

4122:

Wojtowicz, J. Über die korrekte Definition des Ranges eines nomographischen Polynoms und über die Stetigkeit und die Differenzierbarkeit der verallgemeinerten nomographischen Polynome. Ann. Polon. Math. **8** (1960), 177–183.

From the author's introduction: "Der Begriff des Ranges eines nomographischen Polynoms, obwohl in jedem nomographischen Lehrbuch angewandt, ist bisher nicht korrekt definiert worden. Die von verschiedenen Verfassern aufgestellten Definitionen sind ungenau und sogar falsch. In der vorliegenden Arbeit wird eine korrekte Definition des Ranges eines nomographischen Polynoms aufgestellt. Ausserdem enthält die Arbeit Beweise der Sätze, welche in einem gewissen Sinne umgekehrt sind zu bekannten Sätzen über die Summe stetiger Funktionen und über die Differenzierbarkeit der Summe differenzierbarer Funktionen." *S. Kulik* (Long Beach, Calif.)

4123:

Morita, K.; Simokawa, Y. Nomographic representation of the functional relations among three complex variables. Z. Angew. Math. Mech. **40** (1960), 350–359. (German and Russian summaries)

The authors define a Massau's complex chart matrix M_3^c of the third order and show that the functional equation $\det(M_3^c)=0$, including three complex variables z_j $(j=1, 2, 3)$, is expressed by their complex chart in the Gaussian plane, and that an unknown number, one of the z's, is obtained by a pantograph or an elementary geometrical construction. Some examples of these charts are also explained. *S. Kulik* (Long Beach, Calif.)

4124:

Vil'ner, I. A. Nomographing of the generalized function of K. Ya. Zalts without quadratures. Latvijas Valsts Univ. Zinātn. Raksti **28** (1959), no. 4, 131–139. (Russian)

The author generalizes the results given by K. Ya. Zalts in his paper on nomographing the function $F_1K_{23}+F_2L_{31}+F_3M_{12}$ [see Mat. Sb. **33** (**75**) (1953), 383–388; MR **16**, 753], in that he considers abstract variables and does not require differentiability of the function.

S. Kulik (Long Beach, Calif.)

4125:

Kreĭnes, M. A.; Vaĭnštein, I. A.; Aĭzenštat, N. D. Some examples of non-nomogramizable functions. Mat. Sb. (N.S.) **48** (**90**) (1959), 377–395. (Russian)

The authors consider (a) functions which can be nomogrammed on a mesh built in a rectangle and (b) functions that can be nomogrammed in a rectangle by means of a continuous function. They also construct examples of non-nomogramizable functions.

S. Kulik (Long Beach, Calif.)

4126:

Kirkpatrick, E. T. Tables of values of the modified Mathieu functions. Math. Comput. **14** (1960), 118–129.

The Mathieu functions are $\mathrm{Ce}_n(u, q)$ and $\mathrm{Se}_n(u, q)$, the solutions of the equations

$$\frac{d^2y}{du^2}+(a-2q\cos 2u)y=0,$$

$$\frac{d^2y}{du^2}-(a-2q\cosh 2u)y=0.$$

These two equations usually arise in physical problems involving elliptical boundaries.

In this paper the author outlines methods of numerical evaluation of these functions, and discusses a few of the many difficulties which arise in their evaluation. He gives tables of the four functions $\mathrm{Ce}_{2n}(u, q)$, $\mathrm{Ce}_{2n+1}(u, q)$, $\mathrm{Se}_{2n+1}(u, q)$ and $\mathrm{Se}_{2n+2}(u, q)$, over the ranges $q=1(1)10(2)20$, $u=0.1(0.1)1.0$, $n=0, 1, 2$, to 4 or 5 decimals. These tables were computed on an IBM 650 electronic computor. *L. J. Slater* (Cambridge, England)

COMPUTING MACHINES

See also 4326, 4590.

4127:

Lehmer, D. H. Teaching combinatorial tricks to a computer. Proc. Sympos. Appl. Math., Vol. 10, pp. 179–193. American Mathematical Society, Providence, R.I., 1960.

Starting from the simplest programme for determining if x belongs to the set of n numbers $a_1, a_2, \cdots, a_n$ when no ordering relationship exists among the a_r, other programmes are described which reduce the average number of search operations from $n/2$ to $\log_2 n$. These simple considerations lead to an analysis of computer methods for handling permutation problems. The methods of Paige-Tompkins, Marshall Hall, D. N. Lehmer, and Walker are discussed and useful numerical estimates of running time are provided.

The paper is an excellent one from the expository point of view but the future prospects of computers in this field are well summarized in the concluding sentence: "Whatever improvements are made, however, one has only to increase n a little and obtain a hopeless problem about permutations." *A. D. Booth* (London)

4128:

Šileĭko, A. V. Digital models. Avtomat. i Telemeh. **20** (1959), 1687–1697 (Russian. English summary); translated as Automat. Remote Control **20**, 1638–1649.

Authors' summary: "A systematic survey is made of contemporary literature dealing with a class of new computing devices, namely, digital differential analyzers, digital computers operating by the method of summing increments, and with individual function generators whose input and output quantities are presented by the method of delta-modulation of pulsed electrical voltages. The theoretical basis of the operation of digital models is formulated, and comparisons are made of their characteristics with the characteristics of electronic digital computers and of analog computers. For this, a computing machine is considered as an equivalent filter which transforms an input signal, and as the criterion for comparing different classes of machines, we take the product of the filter's resolving power by the width of the band-pass spectrum."

4129:

Wang, Hao. Toward mechanical mathematics. IBM J. Res. Develop. **4** (1960), 2–22.

The author proposes that a new branch of applied logic be recognized which may be called "inferential analysis". Inferential analysis is to assist in the construction of proofs for theorems just as numerical analysis assists in calculations. It is the availability of the high speed calculating machines that makes the present time ripe for the development of inferential analysis.

To illustrate the feasibility of using machines to prove theorems, several of the author's experiments with theorem-proving on an IBM 704 machine are described. The first axiomatic system P considered is a variant of Gentzen's sequent calculus formulation of the classical propositional calculus. [Gentzen's calculus is well described in S. C. Kleene, *Introduction to metamathematics*, Van Nostrand, New York, 1952; MR **14**, 525.] A combined proof and decision procedure for P provides, from a formula, one or more sequents in which no logical connectives occur, and then tests these sequents. A proof of the consistency and completeness is provided for a variant of P in which the only connective is the stroke. P is also extended to the system P_e, the propositional calculus with equality.

A combined proof and decision procedure is also provided for a system Q_p for the AE predicate calculus with equality; that is, the procedure is defined for only those formulae which have a prenex normal form in which no existential quantifier precedes a universal quantifier. The procedure does not require that the formulae be in prenex normal form but rather puts the formulae into what the author calls miniscope form; the form in which each quantifier occurring has the smallest possible scope. Q_p is not complete—there exist theorems of the predicate calculus with an AE prenex normal form which are not provable in Q_p—but two modifications of Q_p, Q_q and Q_r, are discussed which are stated to be complete.

Finally two systems Q and Q' are discussed which are formulations of the whole predicate calculus with equality.

Results of three machine programs are described. Programs I and III are realizations of the proof and decision procedures for P_e and Q_p respectively. Program II was written to select "interesting" propositional theorems. Program I is able to produce proofs for 220 propositional theorems of *Principia Mathematica* in less than 3 minutes of actual calculating time. Program II was not very successful. It is stated that one form of Program III can produce proofs in a "reasonably short" time for 139 theorems in the predicate calculus with equality taken from *Principia*.

In a lengthy conclusion the author mentions areas in which theorem-proving machines might be expected to assist the theoretical investigations of mathematicians.

P. C. Gilmore (Yorktown Heights, N.Y.)

4130:

Ershov, A. P. ★Programming programme for the BESM computer. Translated from the Russian by M. Nadler. Edited by J. P. Cleave. Pergamon Press, London-Oxford-New York-Paris, 1959. vi+158 pp. $10.00.

The translation of this from the original Russian can be considered a contribution to the art of writing compilers. The translation should have come earlier. Since PP-2, the compiler in question, was in operation in 1956, available description of its detailed structure at the present time does not describe the present state of Soviet compiler writing (for example, 12 men assigned to construct an ALGOL translater for the new Russian Academy of Sciences Computation Center at Novosibirsk, under the author's direction). The reader who expects to learn the secret of compiler writing will be disappointed. On the other hand, the reader who is interested in seeing the result of a development in this area almost entirely unaffected by Western work (only Rutishauser's 1952 paper is quoted) will find his time spent in reading large portions of this book well spent.

The compiler described was not the first, it is stated, constructed in the Soviet Union. A previous one had

grown out of Professor Lyapunov's seminar on programming at Moscow State University. Lyapunov's "operational method" has been the basis for these two and other investigations, and is explained thoroughly in this book. The "operational method" might be described as use of an abbreviated algebraic compiler language, with single elements describing individual subroutines. The source language, then, is much like IT, FORTRAN, and ALGOL, with some symbols available that are not in these U.S. languages. The question of character set input to the computer is avoided completely by the simple act of encoding each symbol into an integer, and then entering the integers in sequence.

The target language is that of the BESM, an at-least-four-year-old, three-address-instruction computer, mainly used for scientific calculations, without adequate secondary storage at the time of writing.

The most interesting ideas described are the following. (1) A more concise way of describing transfers of control dependent on a variable x then available in any western language, including ALGOL. (2) Optional use of machine language (only recently adopted in U.S. compilers). (3) The well-known "Ershov Algorithms" for minimization of temporary storage and elimination of repeated terms in substitution statements.

Programming of loops in PP-2, as well as handling of indices, has apparently no more generality than that of FORTRAN.

The entire PP-2 compiler is described, almost instruction by instruction, and a detailed reading of some of its pieces could be of value to a compiler-writer looking for new tricks. A description of the BESM–1 included in the book may be of interest to persons interested in comparative equipment as of 1956.

The translation is a good one, and credit should probably be given to the editor for making terminology agree in general with standards in the U.S. and the United Kingdom. Minor errors in the photographically reproduced typescript are not as important in light of the book's overall interest. *J. W. Carr, III* (Chapel Hill, N.C.)

4131:

Billeter, Ernst P. Über die Bedeutung elektronischer Rechenautomaten in der Statistik. Schweiz. Z. Volkswirtschaft. Statist. **93** (1957), 326–331.

Expository paper. *F. L. Bauer* (Mainz)

4132:

Chandon, Emil C. Möglichkeiten des Einsatzes von elektronischen Rechenanlagen in der amtlichen Statistik. Allg. Statist. Arch. **43** (1959), 348–353.

The author, director of the Land Statistical Office for North Rhine-Westphalia, compares very thoroughly the use of conventional punched card equipment, medium-sized digital computers, and large-sized digital computers in governmental statistical procedures. Surprisingly, to a U.S. reader, his conclusion is that the decision for large rather than medium-sized computers for large-scale data processing (of the type with which he is familiar) should be "carefully and thoroughly evaluated". The author apparently feels that the decentralized data-processing organization with medium-sized machines may be the best solution. This is a very thorough and thoughtful discussion that is worth recommending to a reader interested in making decisions as to size and type of data processing equipment to install.

J. W. Carr, III (Chapel Hill, N.C.)

4133:

Giehl, Rudolf. Die Bedeutung der elektronischen Rechenanlagen für die Weiterentwicklung des statistischen Bearbeitungsprogramms. Allg. Statist. Arch. **43** (1959), 357–362.

This is a non-technical discussion of present and future use of digital computers in statistical analyses. The author gives a simple example solved by hand, punched cards, card calculator, and on a computer. He stresses the increased need for unity in all stages of the problem from data preparation to evaluation of results (the systems approach), and points out the great possible gains in control of data and calculation by programmed consistency and other error checks.

J. W. Carr, III (Chapel Hill, N.C.)

4134:

Greenberg, H. J. Solving structural mechanics problems on digital computers. Structural mechanics, pp. 533–556. Pergamon Press, New York, 1960.

This is a critical survey of applications of digital computers to structural mechanics. It includes brief factual descriptions of typical computing systems, a list of existing codes for basic standard problems, a survey of codes under development for more specific problems, and a detailed discussion of three case histories. These are (i) non-linear deflection of spherical shells, (ii) non-linear bending and buckling of circular plates, and (iii) torsion of cylinders with non-linear stress-strain characteristics. The paper concludes with recommendations for future work. *R. Hill* (Nottingham)

4135:

Samuel, Arthur L. Programming computers to play games. Advances in computers, Vol. 1, pp. 165–192. Academic Press, New York, 1960.

This article is a very clearly written review of some of the programs that have been written to make computers play games. The author describes the feasibility discussions of Shannon, Turing and Strachey, the chess programs of Bernstein, the Los Alamos group, and Newell, Shaw and Simon, and finally, his own work on checkers. It should be noted that programs for two and three move mates in chess ascribed to C. E. Shannon and the reviewer were actually developed by two different groups of M.I.T. students.

J. McCarthy (Cambridge, Mass.)

4136:

Rheinboldt, Werner C.; Menard, John P. Mechanized conversion of colorimetric data to Munsell renotations. J. Opt. Soc. Amer. **50** (1960), 802–807.

Authors' summary: "A program for a high-speed digital electronic computer is described for performing the computation of the Munsell renotations H, V, C corresponding to given CIE chromaticity coordinates x, y and daylight reflectance Y. Mathematically, this is equivalent to a three-dimensional coordinate transformation where two of the three transformation functions are given only

numerically for a grid of discrete points. Since this grid consists of approximately 5000 points which are non-uniformly spaced, the major problem was to devise an economic scanning routine in order to find the point used in the interpolation. This was accomplished by consistent use of vector algebra and the help of an interpretive routine for vector operations."

4137:

Hoen, K.; Grandage, A. H. E. Calculation of inbreeding in family selection studies on the IBM-650 data processing machine. Biometrics **16** (1960), 292–296.

4138:

Supino, Giulio. La teoria dei modelli. Confer. Sem. Mat. Univ. Bari **39–40**, 36 pp. (1958).

The paper has two parts, the first one being an exposition of the classical theory of modeling and dimensional analysis, while the second one deals with analogue models for both ordinary and partial differential equations.

H. Bückner (Madison, Wis.)

4139:

Ostrovskiĭ, G. M. An approximate method for solving equations whose orders exceed the capabilities of analog computers. Avtomat. i Telemeh. **20** (1959), 1129–1130 (Russian. English summary); translated as Automat. Remote Control **20** (1960), 1095–1096.

MECHANICS OF PARTICLES AND SYSTEMS

4140:

Kirgetov, V. I. On "virtual displacements" of material systems with linear differential constraints of the second order. Prikl. Mat. Meh. **23** (1959), 666–671 (Russian); translated as J. Appl. Math. Mech. **23**, 956–962.

In this paper an extension of Četaev's definition [Izv. Kazan. Fiz. Mat. Obšč. (3) **6** (1934), 68–71] of virtual displacements is given, this extension being introduced within the principle of Gauss. Assuming linearity of the proposed definition and independence of virtual displacements on the forces acting on the system, the uniqueness of such a definition is shown.

E. Leimanis (Vancouver, B.C.)

4141:

Kuznecov, B. G. Generalized virtual displacements. Prikl. Mat. Meh. **23** (1959), 672–680 (Russian); translated as J. Appl. Math. Mech. **23**, 963–974.

The author proposes a generalization of the definition of virtual displacements so as to make it applicable to ideal constraints as well as certain other interactions between the particles of a mechanical system and the constraints. The general virtual displacements so defined depend on the law of interaction, and the commonly accepted definition of virtual displacements is obtained as a special case of this law. Some illustrative examples are given.

E. Leimanis (Vancouver, B.C.)

4142:

Legendre, Robert. Répartition des contraintes dans un demi-plan. C. R. Acad. Sci. Paris **250** (1960), 2995–2996.

Author's summary: "Les effets de forces sur la frontière et de forces ou couples concentrés en des points du demi-plan sont successivement étudiés."

4143:

Četaev, N. G. On certain constraints with friction. Prikl. Mat. Meh. **24** (1960), 35–38 (Russian); translated as J. Appl. Math. Mech. **24**, 43–47.

The method of Lagrangian multipliers is applied to non-holonomic systems in which the friction forces do no work. For holonomic systems the results are written in terms of Rayleigh's dissipation function.

P. Franklin (Cambridge, Mass.)

4144:

Albrecht, Felix. Sur la théorie qualitative des systèmes dynamiques à déferlement. Com. Acad. R. P. Romîne **9** (1959), 1135–1139. (Romanian. Russian and French summaries)

Author's summary: "L'auteur présente l'ébauche d'une théorie Poincaré-Bendixson pour les systèmes dynamiques à déferlement dans le plan, engendrés par un nombre fini de systèmes dynamiques classiques, définis dans des domaines homéomorphes à un demi-plan ou à une bande plane."

4145:

Mayer, Arthur. Rotations and their algebra. SIAM Rev. **2** (1960), 77–122.

Finite rotations of a rigid body in three-dimensional space are treated as the elements of the algebra of orthogonal matrices and as an important application the derivation of the equations of motion is given. "There can be no claim to originality in either the results or the methods employed to get them." The paper is very clearly and carefully written and has in the reviewer's opinion much educational value. In I, the elementary theory is developed; the Eulerian angles, e.g., are defined in an instructive way; II also gives an application to resolver systems in analog computers providing the means for the continuous solution in time of problems involving vectors in rotating coordinate systems. III deals with the differential calculus of rotations, IV considers the physical interpretation of the derivatives, and, in V, the equations of motion of a rigid body are derived. Thereby much attention is given to the angular velocity, which behaves like a vector, being essentially a skew-symmetric tensor. The isomorphism between the two is carefully treated in VI by means of the notion of "complementary vectors".

O. Bottema (Delft)

4146:

Grandori Guagenti, Elisa. Un'osservazione sull'attrito dinamico. Boll. Un. Mat. Ital. (3) **14** (1959), 142–150. (English summary)

A plane rigid body B moves on a rough plane, the friction being according to Coulomb's law. Then the frictional force acting on a mass-element P of B is opposite to P's velocity and therefore the resulting

resisting force depends on the motion of B. As an example the author considers a homogeneous ring, which is everywhere in contact with a horizontal plane.

O. Bottema (Delft)

4147:

Timotin, Alexandru. On the definition of the volume density of a force. Bul. Inst. Politehn. Bucureşti **20** (1958), no. 3, 139–147. (Romanian. Russian, English, French and German summaries)

4148:

Chin, Yuan-shun; Liu, Ying-ching. Unconditional stability of systems with time lags. Sci. Record (N.S.) **4** (1960), 79–82.

Consider the vector-matrix system $dx/dt = Ax(t) + Bx(t-\tau)$, $\tau > 0$. If the solution $x = 0$ is stable for all $\tau \geq 0$, the system is said to be asymptotically stable. For systems of dimension one and two, necessary and sufficient conditions for asymptotic stability are given.

R. E. Bellman (Santa Monica, Calif.)

4149:

Liu, Ying-ching. Effects of large time lags on stability of dynamical systems. Sci. Record (N.S.) **4** (1960), 83–87.

Consider the two systems $dx/dt = Ax(t)$, $dx/dt = Ax(t) + Bx(t-\tau)$. In previous work, the author has investigated the connection between the stability of the null solutions of each equation for τ small. In this paper, the question of large τ is considered. The cases of dimensions one and two are studied in detail.

R. E. Bellman (Santa Monica, Calif.)

4150:

Kalinin, S. V. The stability of periodic motions in the case of two zero roots. Prikl. Mat. Meh. **23** (1959), 975–977 (Russian); translated as J. Appl. Math. Mech. **23**, 1393–1397.

The author considers some particular stability criteria for periodic motion in the case where there are two zero characteristic roots. The results are too detailed to summarize. Use is made of the Lyapunov function.

R. E. Bellman (Santa Monica, Calif.)

4151:

Plainevaux, J. E. Mouvement d'un rotor asymétrique tournant dans les paliers élastiques d'une équilibreuse. Z. Angew. Math. Mech. **40** (1960), 359–367. (German, English and Russian summaries)

An unsymmetrical rigid body rotates with constant angular velocity about an axis fixed in the body. The ends of this axis are supported by vertical elastic frames which permit small horizontal displacements at right angles to the axis. Small oscillations (with two degrees of freedom) result from the rotation of the rigid body. This motion is studied in detail in view of its importance to the theory of balancing machines. The equations of motion are linearised in the usual manner, gravity being neglected. These equations contain periodic terms and their approximate solution is obtained by means of perturbation methods. The resulting motion appears to be periodic. The author's method does not, however, lead to the so-called matrix balancing equation [cf. K. Federn, same Z. **25/27** (1947), 164–165].

H. Rund (Durban)

4152:

Marinescu, Matei. Sur certaines propriétés de la "résonance dynamique". Com. Acad. R. P. Romîne **9** (1959), 783–790. (Romanian. Russian and French summaries)

Author's summary: "L'auteur part d'un travail antérieur, sur la 'résonance dynamique' dans des circuits série à inductance variable, et établit la méthode de calcul permettant de résoudre effectivement la résistance et l'inductance dynamique ainsi que l'influence exercée sur ces dernières par un désaccord—positif ou négatif—entre la valeur de la capacité et la valeur pour laquelle la 'résonance dynamique' a lieu. Il établit ainsi qu'une faible variation du désaccord—de sens positif ou négatif—change le régime de fonctionnement du système: de moteur en générateur ou vice-versa."

4153:

Lancaster, P. Free vibrations of lightly damped systems by perturbation methods. Quart. J. Mech. Appl. Math. **13** (1960), 138–155.

Perturbation theory is used to develop a method of estimating the frequencies, rates of decay and modes of vibration of lightly damped systems; the method requires the knowledge of the natural frequencies and principal modes of the corresponding undamped system. Special methods are developed for systems in which some of the natural frequencies are (a) exactly equal and (b) nearly equal.

G. B. Warburton (Edinburgh)

4154:

Grobov, V. A. On the construction of asymptotic approximations describing non-stationary processes in nonlinear gyroscopic systems with the aid of the averaging principle. Ukrain. Mat. Ž. **11** (1959), 213–216. (Russian)

4155:

Panteleev, V. L. Influence of the curvature of the blade on the forced oscillations of a pendulum of long period. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 3, 47–53. (Russian)

4156:

Karasev, M. D.; Suhov, A. M. Even harmonics of nonlinear oscillation contour with odd non-linearity. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 4, 123–129. (Russian)

4157:

Minorsky, Nicolas. Sur l'interaction des oscillations non-linéaires. C. R. Acad. Sci. Paris **250** (1960), 3103–3105.

The author has previously [same C. R. **234** (1952), 292–294; MR **13**, 844] studied solutions of the equation $\ddot{x} + \epsilon(x^2 - 1)\dot{x} + [1 + (a - cx^2)\cos 2t]x = 0$ by means of the heuristic stroboscopic method. The study is continued in the present note, and some further results relating to the existence and stability of periodic solutions are obtained. It should be noted that the title is merely conventional, and that it does not suggest the actual contents of the note.

L. A. MacColl (New York)

4158:

Minatani, Tomotada. On the solution of some nonlinear forced vibrations. II. Sci. Rep. Kagoshima Univ. No. 7 (1958), 1–10.

[Part I in same Rep. No. 6 (1957).] The forced oscillations of a system of one degree of freedom, linear restoring force, and Coulomb friction are studied.

J. P. LaSalle (Baltimore, Md.)

4159:

Barbălat, I. Oscillateur non linéaire autonome, avec toutes les solutions périodiques. Com. Acad. R. P. Romîne **9** (1959), 691–696. (Romanian. Russian and French summaries)

Author's summary: "Pour un équation des oscillations non linéaires autonomes on établit des conditions suffisantes pour que toutes les solutions, quelles que soient leurs conditions initiales, soient périodiques."

4160:

Bordovskiĭ, P. V. Horizontal motion of a point of variable mass in a resistance medium under a linear law of resistance. Odessk. Vyšee Inž. Morsk. Učil. Nauč. Trudy No. 3 (1958), 194–198. (Russian)

STATISTICAL THERMODYNAMICS AND MECHANICS

See also 4354, 4356, 4358, 4359.

4161:

Turner, R. E. Temporal fluctuations in a classical linear system. Physica **26** (1960), 274–283.

The author deals with the temporal evolution of a classical coupled linear system (i.e., with a Lagrangian quadratic in velocities and displacements) in which the initial values are chosen from a canonical ensemble. The later distributions for squared displacements and velocity autocorrelation functions are obtained. These are expressed in terms of the frequency spectrum. The author discusses the Langevin equations for the system in what appears to be an unnecessarily restrictive manner.

G. Weiss (Washington, D.C.)

4162:

Turner, R. E. Motion of a heavy particle in a one dimensional chain. Physica **26** (1960), 269–273.

An expression is given for the velocity autocorrelation function for a heavy particle in a one dimensional chain. The author derives the phenomenon of seeming irreversibility from his expression, a result which is well known on more general grounds. His discussion of the velocity autocorrelation involves steps of dubious validity with regards to limit taking. *G. Weiss* (Washington, D.C.)

4163:

Enoch, Jacob. Equivalence of the Landau and Fokker-Planck collision terms. Phys. Fluids **3** (1960), 353–354.

Author's summary: "It is shown that the Landau collision term is equivalent to the Fokker-Planck type collision term as derived on the basis of the assumption that binary collisions predominate."

W. P. Allis (Cambridge, Mass.)

4164:

Almazov, A. B. The spectrum of an ideal Fermi gas in a lattice. Dokl. Akad. Nauk SSSR **131** (1960), 529–531 (Russian); translated as Soviet Physics. Dokl. **5**, 304–306.

4165:

Kraichnan, Robert H. Condensation of an imperfect boson gas. Phys. Fluids **2** (1959), 463–465.

Author's summary: "The nature of the Einstein-Bose condensation in an imperfect boson gas is discussed. It is argued that no matter how low the temperature, provided it is not zero, macroscopic second-sound waves should be weakly excited as thermal fluctuations about equilibrium. This implies that the single-particle momentum distribution must be smeared so that the state of zero momentum cannot contain a finite fraction of the particles. A microscopic mechanism which independently leads to a momentum spread is discussed. It is suggested that the actual Einstein-Bose condensation of an imperfect gas takes place in energy rather than momentum but that even in terms of this variable it becomes perfectly sharp only at zero temperature. This hypothesis is formulated analytically in terms of correlation functions of the second-quantized boson field."

E. J. Verboven (Utrecht)

4166:

Zastavenko, L. G. A method for the calculation of phase volumes. Ž. Èksper. Teoret. Fiz. **37** (1959), 1319–1323 (Russian. English summary); translated as Soviet Physics. JETP **10** (1960), 939–942.

Author's summary: "A method for calculating phase volumes is developed for 2, 3, 4, and 5 particles. The error of the method increases with the number n of particles, but evidently does not exceed 5 percent for $n=5$."

4167:

Zel'dovič, Ya. B.; Rabinovič, E. M. Conditions for the applicability of statistical formulas to a degenerate Fermi gas. Ž. Èksper. Teoret. Fiz. **37** (1959), 1296–1302 (Russian. English summary); translated as Soviet Physics. JETP **10** (1960), 924–928.

Authors' summary: "A degenerate ideal Fermi gas in an arbitrary potential field is considered. It is shown that a sufficient condition for the applicability of statistical formulas to the problem of the change of the density under the action of the potential $V(r)$ is that the motion of the particles with the maximum (Fermi limit) energy be a quasiclassical motion. This result is not invalidated by nonapplicability of the quasiclassical approximation to the motion of particles with smaller energies, and in particular, for $V<0$, to that of bound particles. The corrections to the statistical formulas in the one-dimensional and three-dimensional problems have opposite signs."

4168:

Strel'cova, E. A. Non-stationary processes in electrolyte theory. Ukrain. Mat. Ž. **11** (1959), 83–92. (Russian. English summary)

4169:

Evseev, A. M. Radial distribution of atoms in a liquid. Dokl. Akad. Nauk SSSR **131** (1960), 789–792 (Russian); translated as Soviet Physics. Dokl. **5**, 333–336.

4170:

Mott-Smith, H. M. Collision trajectories for inverse power intermolecular potentials. Phys. Fluids **3** (1960), 721–724.

Author's summary: "If $-cr^{-n}$ is the intermolecular potential for separation r, b the classical collision parameter, μ the reduced mass, g the relative velocity before collision, and χ the angle of deflection of the relative velocity due to collision, it is found that

$$\chi = -\sum_{t=1}^{\infty} \frac{\Gamma(\frac{1}{2}nt+\frac{1}{2})\Gamma(\frac{1}{2})}{t!\Gamma(\frac{1}{2}nt-t+1)}\left[\frac{2c}{\mu g^2 b^n}\right]^t,$$

the series converging only for $b>b_0$, where $\mu g^2 b_0^n = \frac{1}{2}n^{n/2}(n-2)^{1-n/2}$, this being true both for c positive (attractive potential) and c negative (repulsive potential). This result makes it possible to calculate, by analytic integrations, the contributions to the elementary collision integrals from the range $b>b_0$. It also simplifies greatly the calculation of the collision integrals for both attractive and repulsive inverse power potentials which can be evaluated entirely by analytic means, for $n>2$."

4171:

Dahler, John S.; Cohen, E. G. D. A cell-cluster theory for the liquid state. VI. Binary liquid solutions and hole theory. Physica **26** (1960), 81–102.

[For part V see Cohen and Rethmeier, Physica **24** (1958), 959–969.]

Authors' summary: "The formal cell-cluster theory for a binary liquid solution is given. The configurational partition function is expanded in terms of cluster integrals which are small if the molecules are mainly confined to the lattice sites of a virtual lattice system. The evaluation of the partition function is performed in the quasi-chemical as well as in the Bragg-Williams approximation. The methods used are very similar to those employed by Guggenheim in his studies on polymer solutions. When only single cells are taken into account a generalization of the partition function used by Prigogine and his coworkers is obtained. The corrections due to the introduction of clusters of two cells are given and applied to a few simple cases.

"The same formalism can be used to construct the cell-cluster theory for a liquid with a fixed number of holes. The necessary modifications are indicated and the results given for the case that only clusters of one and two cells are considered. As an example a one dimensional ideal gas with holes is treated."

4172:

Stillinger, Frank H., Jr. Approximations in the theory of dense fluids. Phys. Fluids **3** (1960), 725–732.

Author's summary: "A fluid of rigid spheres in equilibrium is considered from a viewpoint which allows the deduced equation of state to reflect very sensitively the accuracy of two approximations to the triplet distribution function. Specifically, these approximations are: (1) the usual Kirkwood superposition scheme, and (2) assumption that the correlation of excess particles near a fixed particle pair is additively composed of the excesses induced individually by each member of the pair (linear correlation field hypothesis). Granted only these hypotheses, each in turn, the rigorous statistical mechanical relations between rigid-sphere distribution functions and the thermodynamic pressure and compressibility lead unambiguously to nonlinear differential equations for the pressure as a function of density. The simply obtained numerical solutions clearly demonstrate that assumption (1) is considerably superior to (2)."

4173:

Lebowitz, Joel L. Asymptotic value of the pair distribution near a wall. Phys. Fluids **3** (1960), 64–68.

The asymptotic behaviour of the pair distribution function of a fluid is calculated when one particle is in contact with the wall of the container and the other is in the interior of the fluid. It is found to be different from the asymptotic form of the pair distribution function when both particles are in the interior of the fluid; the compressibility is replaced by the inverse of the pressure. The expression is obtained by using the virial theorem for fluctuations in total momentum and making the assumption that there is no long range order in the fluid. The result is used to derive the second virial coefficient and to derive the equation of state of a one-dimensional hard sphere gas. A brief discussion of quantum systems is given.

D. J. Thouless (Birmingham)

4174:

Weinberg, Alvin M.; Wigner, Eugene P. ★The physical theory of neutron chain reactors. The University of Chicago Press, Chicago, Ill., 1958. xii + 801 pp. $15.00.

This is a most comprehensive treatise on nuclear reactor theory and the underlying physics. Being comprehensive it is addressed primarily to the reader who already knows something about the reactors rather than to a complete newcomer in the field. At the same time it is addressed more to the people directly concerned with selecting and design of a nuclear reactor, the nuclear engineer and the experimental physicist, rather than to a theoretical physicist or a mathematician.

The book is divided into four parts. After an introductory chapter where the main nuclear physics and engineering problems facing reactor designer are indicated, the rest of part I (chapters 2 to 6) is devoted to nuclear properties of various substances and to nuclear reactions. The general background of nuclear physics is kept to a minimum (compatible with clear understanding of practical implications), while the main attention is given to those reactions and properties that are of direct importance for the reactors, such as fission, neutron absorption and neutron scattering. The main purpose is to allow the reader to extrapolate from the substances whose nuclear properties are well known to those whose nuclear properties are not so well known. Part II (chapters 7 to 11) deals with neutron diffusion, slowing down and multiplication in bulk media. Both the exact and the approximate equations governing these processes are given, as well as a general discussion of the methods of their solution. In part III the results of the first two parts

are applied to a detailed study of reactors whose core is homogeneous, and in part IV to the reactors whose core has lattice structure. Many of the calculations are compared with the available experimental results.

The authors take throughout a very broad approach, paying as much attention to compact, high-grade-fuel reactors as to more conventional, large, low-grade-fuel ones, as much attention to the special purpose reactors as to more frequent power-and-plutonium-producers, and so on. These features make the book very valuable indeed to a reader who, knowing something about the reactors, wants to deepen his understanding; or who, being familiar with one type of reactors is turning his attention to other types. But the very abundance of detail makes this book hardly suitable for a beginner. Also, the qualitative discussions preceding the detailed mathematical analysis are, at times, not fully understandable without some prior knowledge of reactor theory. This is so, for instance, in the chapter on reactor kinetics.

Each specific calculation on the nuclear reactors involves essentially two stages. On one hand one has to round up the relevant nuclear data, supplementing it, if necessary with some educated guesses, carry out some smoothing and averaging if the method of calculation cannot handle the data in all its complexity, and so on. On the other hand there is mainly calculation once the data has been collected and pre-digested. The authors' attention is centered on the first stage and this is discussed with extreme thoroughness. But in so far as second stage is concerned the present book does not go much beyond the much more elementary books [such as Glasstone and Edlund, *The elements of nuclear reactor theory*, Van Nostrand, New York, 1956]. Only the simplest approximations—the diffusion approximation, the age-velocity theory, etc. are described in detail (and, for a theoretician, perhaps in unnecessary detail). The account of the more advanced general methods, such as spherical harmonics methods, Carlson's S_n methods, etc., is rather sketchy, while many other methods are merely mentioned by name. However, this is not a criticism. There are other books specially devoted to the advanced mathematical methods used in reactor theory. The real need had been for a book giving a thorough appraisal of various physical assumptions used in calculations, and the present book amply meets this need.

While primarily a physicist's book, it does contain some arguments of appreciable mathematical elegance, for instance, their treatment of the "second fundamental theorem of reactor theory". The terminology is on the whole clear, though some innovations in it may be rather controversial. For instance one may feel that introducing the term "Fermi" instead of "barn" for the unit of area, and discarding the term "Fermi" for the unit of length might have been ill-advised. A mathematical reader may be taken aback when the authors say "essential singularity" while they really mean "logarithmic singularity" (p. 240), but this is the only inaccuracy in mathematical terminology that the reviewer noticed. There are very few misprints and the text is abundantly illustrated with tables and diagrams. *B. Davison* (Toronto, Ont.)

4175:

Kihara, Taro; Taylor, Marion H.; Hirschfelder, Joseph O. Transport properties for gases assuming inverse power intermolecular potentials. Phys. Fluids **3** (1960), 715–720.

Author's summary: "The integrals required for the calculation of transport properties for gases are carefully examined, assuming that the intermolecular potentials vary inversely as a power of the separation, $\Phi = -c/r^n$. When the potential corresponds to mutual attraction, the behavior at the origin, corresponding to the centers of the two molecules colliding, must be prescribed because some trajectories of the relative motion reach the origin. Calculations are made on the basis of three models: (a) A rigid core corresponding to the Sutherland potential in the limit that the rigid core approaches zero (for spherical models this is the most realistic model); (b) a transparent core model which corresponds to the limiting behavior of a well-shaped potential; and (c) a random-scattering core model which is an appropriate idealization for molecules without spherical symmetry. The behavior of the collision integrals is considered for the full range of potentials as n goes from one to infinity (or from the Coulomb to the rigid-sphere potential)."

4176:

Virkkunen, Jouko. Neutron thermalization: asymptotic spectrum and diffusion length in a heavy gas with arbitrary absorption. Ann. Acad. Sci. Fenn. Ser. A. VI No. 43 (1960), 27 pp.

4177:

Greene, R. F. Surface transport theory. Phys. and Chem. Solids **14** (1960), 291–298.

4178:

Matsubara, Takeo; Blatt, John M. Bose Einstein condensation of correlated pairs. Progr. Theoret. Phys. **23** (1960), 451–474.

The phenomenon of condensation of correlated pairs of fermions is studied by evaluating the grand partition function using an approximation which is valid also in the condensation region. The relation between the results of the present paper and of earlier work by Blatt and co-workers [see Schafroth, Butler and Blatt, Helv. Phys. Acta **30** (1957), 93–134; MR **19**, 486; Blatt and Matsubara, Progr. Theoret. Phys. **20** (1958), 781–783], is discussed. The case where the "size" of the pair is large compared to the average distance between the pairs is discussed as well as the simpler case where the opposite is true. The first case is important in the theory of superconductivity where the size of the pairs will be of the order of the Pippard coherence length. This paper is to a certain extent a justification of the basic assumption of the Bardeen-Cooper-Schrieffer-Bogolyubov theory of superconductivity, namely, the assumption that the Cooper pairs which lead to superconductivity have zero total linear momentum. *D. ter Haar* (Oxford)

4179:

Meixner, J. Relaxationserscheinungen und ihre thermodynamische Behandlung. Nederl. Tijdschr. Natuurk. **26** (1960), 259–273.

4180:
Balian, Roger; De Dominicis, Cyrano. Expression des grandeurs thermodynamiques en termes d'une fonction de distribution de quasi-particules. C. R. Acad. Sci. Paris **250** (1960), 3285–3287.

Authors' summary: "On exprime, pour un système de volume infini, les grandeurs thermodynamiques en fonction d'une quantité F_ξ, contenant toute la dépendance en l'activité chimique et la température. L'interprétation de F_ξ comme fonction de distribution de quasi-particules justifie et explicite une hypothèse formulée par Landau pour les liquides de Fermi. L'expression de l'énergie moyenne redonne, comme cas particulier, le développement de Goldstone pour l'énergie de l'état fondamental."

4181:
Balian, Roger; De Dominicis, Cyrano. Expression des grandeurs thermodynamiques en termes d'une fonction de distribution de quasi-particules. C. R. Acad. Sci. Paris **250** (1960), 4111–4113.

Authors' summary: "Méthode de calcul des grandeurs thermodynamiques à densité donnée pour un système anisotrope; cas particulier de la température nulle."

4182:
Aubert, Marius. Sur la représentation géométrique de certains phénomènes irréversibles. Ann. Physique (13) **5** (1960), 1177–1184.

ELASTICITY, PLASTICITY

See also 4134, 4527.

4183:
Fraĭfel'd, S. E. General equations of the theory of deformations of materials. Trudy Har'kov. Inž.-Stroit. Inst. **1** (1957), no. 5, 3–37. (Russian)

Meditations upon the strain-stress relationship in mechanics of deformable solids.

S. Drobot (Notre Dame, Ind.)

4184:
Slobodyanskiĭ, M. G. General and complete solutions of the equations of elasticity. Prikl. Mat. Meh. **23** (1959), 468–482 (Russian); translated as J. Appl. Math. Mech. **23**, 666–685.

This paper aims at extending known theorems concerning the completeness of various solutions (in terms of three harmonic displacement potentials) to the field equations of classical elasticity theory. The author distinguishes between "complete" and "general" solutions without, however, defining sharply the intended distinction. The statements and proofs of the new theorems presented here are similarly lacking in precision. {On p. 477 reference is made to a related previous publication [R. A. Eubanks and E. Sternberg, J. Rational Mech. Anal. **5** (1956), 735–746; MR **18**, 163]. In this connection the author asserts that one of the completeness proofs contained in the paper cited involves the divergent improper integral given by Equation (2.23) of the present paper. He goes on to say that "since the integral . . . is improper", the authors of the earlier paper "in proving their theorem neglected the first few terms of the series expansion" of the corresponding integrand. This curious criticism is somewhat bewildering in view of the fact that the integral (2.23) does not appear in the paper under discussion; the integral which was actually introduced in the proof referred to is obtained from (2.23) by removing the singular portion of the integrand and is convergent.}

E. Sternberg (Providence, R.I.)

4185:
Sternberg, E. On some recent developments in the linear theory of elasticity. Structural mechanics, pp. 48–73. Pergamon Press, New York, 1960.

The paper gives useful summary of some topics in linear elasticity, with particular attention to stress functions, St. Venant's principle, and concentrated loads—all areas in which the author himself has contributed and so is particularly knowledgeable.

C. E. Pearson (Cambridge, Mass.)

4186:
Aleksandrov, A. Ya. Some relations between the solutions of plane and axially symmetric problems in the theory of elasticity and the solution of axially symmetric problems by means of analytic functions. Dokl. Akad. Nauk SSSR **129** (1959), 754–757 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 1378–1382.

The author considers coupled problems of kinds described in title and obtains stress state in one by spatial integration of stress state in other. The method appears to be of limited utility. *C. E. Pearson* (Cambridge, Mass.)

4187:
Teodorescu, Petre P. Sur une représentation par potentiels dans le problème tridimensionnel de l'élastodynamique. C. R. Acad. Sci. Paris **250** (1960), 1792–1794.

The author uses Clebsch and Jacovache-type stress functions to obtain general expressions for stresses (rather than more conventional displacements) in time-dependent elastic problems; method and intent are not straightforward. *C. E. Pearson* (Cambridge, Mass.)

4188:
Ponomarev, K. K. Computation of elastic systems by the method of continuation. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 6, 12–36. (Russian)

4189:
Mizuno, Masao. Problem of large deflection of coiled springs. Bull. JSME **3** (1960), 95–103.

The large deflections of a coiled spring are analyzed, the latter being treated as a bar with varying axial force, flexure and shear rigidities. These rigidities are obtained from *Technische Dynamik, Vol. 1*, by Biezeno and Grammel [Springer, Berlin, 1953; p. 616]. Numerical results are calculated, these being compared with experiment. *H. D. Conway* (Ithaca, N.Y.)

4190:

Bondar', V. D. Certain accurate solutions of the compatibility equations for the deformation tensor components in the case of a simple load. Dokl. Akad. Nauk SSSR **130** (1960), 1218–1219 (Russian); translated as Soviet Physics. Dokl. **5**, 71–73.

The author discusses the problem of satisfying the compatibility conditions for finite elastic deformations. These conditions on the deformation tensor, ε_{ij}, imply that the Riemann tensor of three-dimensional space vanishes. The formulation of these conditions for a simple load by L. I. Sedov [Prikl. Mat. Meh. **23** (1959), 400–402; MR **22** #1211] is used; the coordinate system, x^j, is general curvilinear. Two types of solutions are shown to exist. Both depend on classifying the geometric object

$$G_{jkp} = \frac{\partial \varepsilon_{kj}}{\partial x^p} + \frac{\partial \varepsilon_{pj}}{\partial x^k} - \frac{\partial \varepsilon_{kp}}{\partial x^j}.$$

Since ε_{kj} is a symmetric tensor, G_{jkp} behaves like the Christoffel symbol of the first kind under coordinate transformation. The forms of the corresponding ε_{ij} are indicated. *N. Coburn* (Ann Arbor, Mich.)

4191:

Bufler, H. Bemerkung zur vollständigen Auswertung des Spannungszustandes in Platten mit Hilfe der Spannungsoptik. Ing.-Arch. **29** (1960), 150–152.

4192:

Szelągowski, Franciszek. A plate strip acted on by external load. Rozprawy Inż. **8** (1960), 83–91.

From the author's summary: "The solution of the problem of an isotropic plate strip subject to the action of a load on the upper and lower edge. The solution reduces, in principle, to the determination of two holomorphic functions figuring in the equations for the stresses and the displacements for any point of the region considered."

4193:

Koiter, W. T. Stress distribution in an infinite elastic sheet with a doubly-periodic set of equal holes. Boundary problems in differential equations, pp. 191–213. Univ. of Wisconsin Press, Madison, 1960.

The doubly-periodic stress distribution in an infinite elastic sheet with a doubly-periodic set of equal holes of arbitrary shape is investigated. The problem is solved by suitable modification of Muskhelishvili's method of Cauchy-integrals. A functional equation for one of the two complex stress functions is reduced to an ordinary Fredholm integral equation of the second kind, and the existence of the essentially unique solution of the problem is proved. *A. E. Green* (Newcastle-upon-Tyne)

4194:

Dvořák, Jaroslav. On the distribution of stress near openings. Apl. Mat. **5** (1960), 81–108, 170–195. (Czech. Russian and English summaries)

The paper is quite an exhaustive summary of the latest results in plane problems of elasticity on stress concentration caused by openings and on the effects of reinforcements around such openings.

R. M. Evan-Iwanowski (Syracuse, N.Y.)

4195:

Muki, R. Asymmetric problems of the theory of elasticity for a semi-infinite solid and a thick plate. Progress in solid mechanics, Vol. 1, pp. 399–439. North-Holland Publishing Co., Amsterdam, 1960.

The paper is a review of work, mostly by the present author, on three-dimensional problems connected with the semi-infinite solid and the thick plate. The forces and temperature applied to the solid may be symmetrical or asymmetrical about a line. Integral transforms are used to obtain general solutions which are then adapted to give solutions to several problems, mostly concerned with indentation and thermoelasticity. Little reference is made to papers using other methods of solution, for instance the work of A. E. Green [Proc. Cambridge Philos. Soc. **45** (1949), 251–257; MR **10**, 649] on the indentation of a semi-infinite solid by a slightly tilted flat-ended circular punch. *G. Eason* (Newcastle-upon-Tyne)

4196:

Remnev, Yu. I. Symmetrical deformation of a sphere in the presence of volume dilatation. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 2, 43–47. (Russian)

4197:

Biernawski, Antoni; Grochowski, Bogusław. Minimum potential design of lattice. Rozprawy Inż. **8** (1960), 137–165. (Polish. Russian and English summaries)

4198:

Kyrklund, Harald. Über die Einschätzung von Biegespannungen in gekrümmten Balken. Acta Polytech. Scandinav. No. 274 (1960), 56 pp.

4199:

Petre, Augustin. Les équations du mouvement dans le cas du flambage des barres droites par choc axial. Bul. Inst. Politehn. Bucureşti **20** (1958), no. 3, 189–198. (Russian, English and German summaries)

Author's summary: "Paper presents the equations of axial and lateral motions of beam cross-sections subjected to a low velocity axial impact when the beam begins to buckle under this impact. By means of these relations the case previously dealt with by the same author is verified when the beam mass is not taken into account, and the whole buckling process is also described. Since the strains are known, the stresses may be evidently determined."

4200:

Petre, Augustin. Détermination de l'équation differentielle de la ligne élastique dans le cas des barres droites, sous l'action des charges longitudinales et transversales. Bul. Inst. Politehn. Bucureşti **21** (1959), no. 1, 145–155. (Russian, English and German summaries)

Author's summary: "The differential equations of the center line curve for a deflected beam subjected to given axial and lateral loads are determined, the additional stresses which are due to strains being also taken into

account. As an application the author presents the calculation of buckling loads according to various laws of the axial load along the beam. The calculation of deflection and bending moments of a beam subjected to tension and compression and also to lateral loads, etc., is included."

4201:

Marshall, F. J.; Ludloff, H. F. A note on Timoshenko beam theory. J. Math. Phys. **38** (1959/60), 175–180.

4202:

Kraus, L. Integralgleichungen gerader Kippträger mit dünnwandigen, offenen und quasi-wölbfreien Profilen. Ing.-Arch. **29** (1960), 187–198.

4203:

Netrebko, V. P. A problem of torsion of rods of variable cross-section. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1958**, no. 6, 7–12. (Russian)

4204:

Ganowicz, Ryszard. A plate strip having ribs on one side. Rozprawy Inż. **8** (1960), 323–342. (Polish. Russian and English summaries)

Author's summary: "It is shown that all the mechanical quantities of an isotropic plate strip having ribs on one side may be represented in the form of closed expressions. The solution is obtained with the assumptions and on the basis of the relations obtained by Pflüger [Ing.-Arch. **16** (1947), 111–120; MR **10**, 85]. A system of three equations for the displacements u, v, w of a point of the middle surface obtained on the basis of this paper is reduced to one equation for the displacement function. Assuming the solution in the form of a trigonometric series, expressions are obtained which can be written in a closed form.

"A numerical example is given of which the results are compared with the values obtained from model tests, which are found to be in agreement with the theoretical solution."

4205:

Delchambre, Roger; Janssens, Paul; Vanbeckbergen, Monique. Résolution exacte de l'équation des plaques circulaires à épaisseur linéairement variable. C. R. Acad. Sci. Paris **250** (1960), 4271–4273.

4206:

Ille, V. Déformations et sollicitations dans une plaque plane oblique. Inst. Politehn. Cluj. Lucrări Şti. **1959**, 219–229. (Romanian. Russian and French summaries)

Author's summary: "Dans cet ouvrage on présente une méthode de calcul pour les déformations et les sollicitations d'une plaque plane oblique, actionnée par des charges uniformes distribuées normalement sur son plan, lorsque la plaque est encastrée sur tout le contour. On part de l'équation différentielle des plaques planes notée en coordonnées cartésiennes

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{\rho}{D}$$

et on transcrit cela pour un système oblique de coordonnées u, v, auquel est rapportée la surface médiane de la plaque, obtenant ainsi l'équation différentielle

$$\frac{\partial^4 w}{\partial u^4}-4\cos\alpha\left(\frac{\partial^4 w}{\partial u^3\partial v}+\frac{\partial^4 w}{\partial u\partial v^3}\right)+2(1+2\cos^2\alpha)\frac{\partial^4 w}{\partial u^2\partial v^2}+\frac{\partial^4 w}{\partial v^4}=\sin^4\alpha\,\frac{\rho}{D},$$

connue d'ailleurs dans la littérature.

"L'intégration de cette équation est faite par une méthode approximative variationnelle: la méthode de la réduction à équations ordinaires. On obtient ainsi l'expression approximative de la flèche w et des moments fléchissants et de torsion selon les lignes des coordonnées obliques."

4207:

Aleksandrov, A. Ya. Some relations between the solutions of the plane and axially symmetric elasticity problems for the infinite plate. Dokl. Akad. Nauk SSSR **128** (1959), 57–60 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 1121–1124.

4208:

Hilton, Harry H.; Feigen, Morris. Minimum weight analysis based on structural reliability. J. Aerospace Sci. **27** (1960), 641–652.

A general method is developed in which the cross-sectional areas of the components of a structure are proportioned in such a manner as to achieve minimum over-all weight for a prescribed probability of failure. In general it is found that the heavier components should be designed for higher individual probabilities of failure. Estimates of possible weight reductions are made for certain cases. Many of the assumptions will require further refinement before the method can be used with assurance. In view of the greatly magnified effects of weight in missiles and space vehicles this area of applied mathematics has become very important and it is to be hoped that rapid progress will be made in extending the method to include non-Gaussian probability distributions, inelastic effects, dimensional variations, etc.

F. R. Shanley (Santa Monica, Calif.)

4209:

Kornishin, M. S.; Mushtari, Kh. M. A certain algorithm of the solution of nonlinear problems of the theory of shallow shells. J. Appl. Math. Mech. **23** (1959), 211–218 (159–163 Prikl. Mat. Meh.).

Using an algorithm proposed by Muštari [Izv. Kazan. Filial. Akad. Nauk SSSR. Ser. Fiz.-Mat. Teh. Nauk **1958**, no. 12, 63–67], a study is made of the finite bending of shallow cylindrical panels of rectangular planform under uniform loading. Use is made of the Bubnov-Galerkin method to integrate the strain compatibility and equilibrium equations. Numerical results are presented.

H. D. Conway (Ithaca, N.Y.)

4210:

Levitan, Edwin S. Forced oscillation of a spring-mass system having combined Coulomb and viscous damping. J. Acoust. Soc. Amer. **32** (1960), 1265–1269.

4211:

Rohonyi, V. Nouvelle méthode grapho-analytique de calcul pour la détermination des vibrations propres aux ressorts à lames composées pour tamis vibratoires. Inst. Politehn. Cluj. Lucrari Şti. 1 (1958), 147–155. (Romanian. Russian and French summaries)

Author's summary: "Le calcul du nombre des vibrations propre à un ressort construit en lames ayant une forme symétrique, formé par des lignes droites et courbes, présente de grandes difficultés, car le problème est statiquement indéterminé et demande donc l'application du théorème de Castigliano. Or l'intégration des expressions des travaux partiels de déformation le long d'une courbe composée, combinée avec l'application de la seconde thèse Castigliano, amène à une telle complication du problème que le calcul devient trop complexe pour les ingénieurs praticiens.

"L'auteur présente une méthode graphique pour éviter l'intégration analytique, méthode par laquelle la constante de l'arc correspondante aux forces verticales, respectivement horizontales, peut être calculée très facilement et en peu de temps, car tout le calcul est réduit à 6 planimétrations, au calcul de 6 constantes et de 2 formules finales."

4212:

Young, Frederick J. Family of bars of revolution in longitudinal half-wave resonance. J. Acoust. Soc. Amer. 32 (1960), 1263–1264.

Author's summary: "Expressions for the velocity potential, velocity transformation ratio, node location, resonant length, stresses, and mechanical impedances are derived for a family of tapered bars of revolution in longitudinal half-wave resonance."

4213:

Movčan, A. A. The direct method of Lyapunov in stability problems of elastic systems. Prikl. Mat. Meh. 23 (1959), 483–493 (Russian); translated as J. Appl. Math. Mech. 23, 686–700.

After pointing out that the usual static and dynamic definitions of the stability of an elastic equilibrium are unsatisfactory, the author defines stability in the sense of Liapunov and develops theorems that enable one to apply Liapunov's direct (second) method to the study of the stability of equilibrium states of elastic systems. He illustrates the method by studying the stability of the plane state of an elastic thin plate of infinite length simply supported along two edges.

Liapunov theorems are given for motions defined in a metric space in a form which the author states are more convenient for some applications than Zubov's version [V. I. Zubov, *Metody A. M. Lyapunova i ih primenenie*, Izdat. Leningrad. Univ., Moscow, 1957; MR 19, 275].

J. P. LaSalle (Baltimore, Md.)

4214:

Budiansky, Bernard; Weinitschke, Hubertus. On axisymmetrical buckling of clamped, shallow, spherical shells. J. Aerospace Sci. 27 (1960), 545–546.

This note reports the close agreement of two independent numerical determinations by the authors of what they define to be the "buckling-pressure" for axisymmetric deformations of clamped shallow spherical shells. Since the reported results do not agree with experiments and other theoretical calculations it is concluded that unsymmetric deformation theories are required to explain the experiments and that the other theoretical results are wrong. However, these conclusions and the validity of the definition of buckling-pressure used are open to serious doubt. In fact some of the calculations claimed to be in error [H. B. Keller and E. L. Reiss, same J. 26 (1959), 643–652; MR 21 #6808] agree quite well with a significant range of the experiments and employ a different definition of buckling pressure based on the existence of nonunique solutions of the boundary value problem.

H. B. Keller (New York)

4215:

Tarasenko, I. I.; Tarasenko, E. N. Conditions of brittle strength of isotropic materials. Zap. Leningrad. Gorn. Inst. 36 (1958), no. 3, 146–155. (Russian)

4216:

Chudzikiewicz, Andrzej. The influence of the deformability of cross-section on the critical force of torsional buckling of a double-tee bar. Rozprawy Inż. 8 (1960), 177–186. (Polish. Russian and English summaries)

4217:

Chudzikiewicz, Andrzej. Bending and flexural-torsional buckling of a channel with deformable cross-sections. Rozprawy Inż. 8 (1960), 253–271. (Polish. Russian and English summaries)

4218:

Kumar, Sudhir. Edge waves in plates. International symposium on stress wave propagation in materials, pp. 119–131. Interscience Publishers, New York, 1960.

The plane stress problem of wave propagation in a strip is considered. The frequency equation is obtained but not solved.

G. Eason (Newcastle-upon-Tyne)

4219:

Mindlin, R. D.; McNiven, H. D. Axially symmetric waves in elastic rods. J. Appl. Mech. 27 (1960), 145–151.

This paper extends earlier work of Mindlin and Hermann [Proc. First U.S. National Congr. Appl. Mech. Chicago, 1951, pp. 187–191, A.S.M.E., New York, 1952] on the derivation of approximate one-dimensional equations for axially symmetric motion of elastic rods of circular cross-section. The equations take into account the coupling between longitudinal, axial shear, and radial modes. The results are compared with the predictions of the exact Pochhammer treatment derived from the three-dimensional equations.

H. Kolsky (London)

4220:

Adachi, Ryuzo. On the wave motion propagated along an elastic cylinder with infinite length. Kumamoto J. Sci. Ser. A 4, 134–143 (1959).

This paper is concerned with the propagation of a train of harmonic waves along an elastic cylinder of

circular section. The main results of the classical theory of Pochhammer and Chree are re-derived, and the dependence of the velocity c of the fundamental longitudinal wave upon a/λ is then discussed (a being the radius of the cylinder and λ the wavelength). As $a/\lambda \to 0$ then, as is well known, $c \to (E/\rho)^{1/2}$, where E is the Young's modulus and ρ the density of the material. The author infers from numerical examples that as $a/\lambda \to \infty$, then $c \to c_R$, the velocity of Rayleigh waves in the medium. This result is also known [D. Bancroft, Phys. Rev. (2) **59** (1941), 588–593].

No references to the literature of this thoroughly investigated problem are provided.

P. Chadwick (Sheffield)

4221:

Mitra, M. Exact solution of the buried line source problem for a uniformly moving line source. Bull. Calcutta Math. Soc. **51** (1959), 109–115.

The plane strain problem of a line source buried in a semi-infinite solid which is suddenly generated and then moves with uniform velocity parallel to the surface of the solid is considered. A method based on work by Cagniard [*Réflexion et réfraction des ondes seismiques progressives*, Gauthier-Villars, Paris, 1939] and Garvin [Proc. Roy. Soc. London. Ser. A **234** (1956), 528–541; MR **17**, 1158] is used to obtain a solution in closed form.

G. Eason (Newcastle-upon-Tyne)

4222:

Mitra, M. Solution of the buried source problem for an extended two-dimensional source in an elastic medium. Proc. Nat. Inst. Sci. India. Part A **25** (1959), 236–242.

Results due to Garvin [cf. #4221] for a line source buried in a semi-infinite solid are extended to the case in which the source is distributed over an area.

G. Eason (Newcastle-upon-Tyne)

4223:

Barenblatt, G. I. The formation of equilibrium cracks during brittle fracture. General ideas and hypotheses. Axially-symmetric cracks. Prikl. Mat. Mech. **23** (1959), 434–444 (Russian); translated as J. Appl. Math. Mech. **23**, 622–636.

The author discusses an extension of Griffith's theory in terms of Sneddon's treatment of the distribution of normal displacements of the surface of a round crack in an infinite elastic solid. Most of the paper is concerned with the formation and growth of axially symmetric cracks when stresses are either applied externally or produced within the brittle solid. Relations are derived between the radius of curvature at the end of the crack and the applied stresses. *H. Kolsky* (Providence, R.I.)

4224:

Ivlev, D. D. The theory of fracture of solids. Prikl. Mat. Meh. **23** (1959), 618–624 (Russian); translated as J. Appl. Math. Mech. **23**, 884–895.

This paper reviews the triaxial stress conditions which lead to failure and discusses the conditions under which fracture and plastic flow can occur. The author suggests that such concepts as discontinuous solutions, possible stress fields and velocity fields as well as extremum theorems can be adapted from the theory of plasticity to the theory of fracture. *H. Kolsky* (Providence, R.I.)

4225:

Bagdoev, A. G. Propagation of pressure in an elastic half-space. Akad. Nauk Armyan. SSR. Dokl. **28** (1959), 49–52. (Russian. Armenian summary)

A plane problem of propagation of a variable pressure applied at the boundary of an elastic half-space is investigated. The boundary of the half-plane is assumed to be invariable in time, and the sound velocity constant. The distribution of pressure along the wave front has been found using the so-called ray method. The equation of the front has been found using Huygens' construction of the envelope of disturbances.

J. Nowinski (Austin, Tex.)

4226:

Tobe, Toshimi. Transverse impact on beams. Bull. JSME **3** (1960), 287–291.

Author's summary: "Problems of lateral vibrations and the dynamic loads produced in a beam struck by a falling ball have been treated by many authors; but in their studies only the impact caused by a sphere moving at a given velocity was discussed. The problem of impact by a falling body with a cylindrical contact surface has not been treated for the reason of its complexity. The relative approach in a cylinder contact has a complicated mathematical form as compared with the case of a sphere-contact, but it is a very important problem in practice. In this paper two methods are developed to treat the problem: (a) step-by-step integration used by Timoshenko and (b) collocation method. The collocation method leads to a reasonable and sufficiently approximate solution without difficulty. Numerical examples and the effects of non-dimensional characteristic coefficients (k_1, k_2, k_3, and k_M) are discussed and the time dependency of deflection of the beam is given in order to compare the impact of a sphere."

4227:

Gazis, D. C.; Herman, R.; Wallis, R. F. Surface elastic waves in semiconductors. Phys. and Chem. Solids **14** (1960), 268–270.

4228:

Savin, G. N.; Ševelo, V. N.; Kužil, A. I. On longitudinal vibrations of an elastic-viscous thread of variable length with a load at an end. Akad. Nauk Ukrain. RSR. Prikl. Meh. **1** (1955), 328–334. (Ukrainian. Russian summary)

4229:

Kužil, A. I.; Ševelo, V. N. Investigation of the effect of imperfect elasticity on the dynamic pull in a thread of variable length with the aid of the asymptotic method. Akad. Nauk Ukrain. RSR. Prikl. Meh. **1** (1955), 41–50. (Ukrainian. Russian summary)

4230:

Neronov, N. P. A note on determining tensions in hoisting ropes. Zap. Leningrad. Gorn. Inst. **36** (1958), no. 3, 94–100. (Russian)

4231:

Severov, Yu. I. Some notes on the dynamics of a solid body and its application to the theory of crushers. Zap. Leningrad. Gorn. Inst. **36** (1958), no. 3, 106–111. (Russian)

4232:

Nowacki, Witold. Stress propagation in an infinite viscoelastic body produced by a time-variable point force. Arch. Mech. Stos. **11** (1959), 737–750. (Polish and Russian summaries)

The stresses produced in an infinite visco-elastic solid by time-dependent body forces are considered. The method of solution is that of multiple Fourier transforms together with a Laplace transform in the time variable. A complete solution is obtained for a body whose stress-strain relationship is of a type proposed by Biot [J. Appl. Phys. **25** (1954), 1385–1391]. This method could be used for other types of solid.

G. Eason (Newcastle-upon-Tyne)

4233:

Morland, L. W. The propagation of plane irrotational waves through an elastoplastic medium. Philos. Trans. Roy. Soc. London. Ser. A **251** (1959), 341–383.

In the last two decades, physical phenomena associated with the propagation of stress waves beyond the limit of linear elastic behaviour in metals (in polycrystalline form) have been given much and increasing attention, from both experimental and theoretical viewpoints. The literature on this subject is extensive, but two important general references are H. Kolsky [*Stress waves in solids*, Clarendon, Oxford, 1953] and J. S. Rinehart and J. Pearson [*Behavior of metals under impulsive loads*, The American Society for Metals, Cleveland, Ohio, 1954]. The intrinsic difficulties in obtaining accurate physical data and in furthering the development of realistic theoretical studies are well known to research workers in this field. A situation of much practical interest is that ensuing when anelastic stress waves are generated under conditions of extremely high pressure and high rate-of-strain, such as occur, for example, with surface loadings generated by high-speed impact between a projectile and a target and by an explosion at the surface of a target. Under such circumstances, for example, peak pressures of order 300 kilobars may be achieved in times of order 10^{-7} sec. and there occur rates-of-strain of order 10^{6} sec.$^{-1}$; and then the dependence of elastic constants upon pressure and of plastic yielding upon rate-of-strain markedly affects the propagation of stress waves occurring in the target. For such situations, realistic constitutive physical equations—one of the basic ingredients involved in theoretical studies—are undoubtedly very complex and the features exhibited can show important differences from one metal to another. The surface loadings are not known simply from the initial conditions but depend upon the details of the process of interaction between the target and the load-producing agent; however, in certain cases, quite good estimates of these loadings (or of their impulses) appear to be possible. It should also be noted that the presence of free boundaries to the target (as is normally the case under practical conditions) can lead to complications in the stress-wave pattern and, for example, material fracture may occur under reflected tensile stresses.

In any detailed quantitative study of non-linear stress-wave propagation in solids, a severe compromise is normally necessary between physical realities and mathematical complexities. Certainly, under conditions extending to high pressure and high rate-of-strain no very great quantitative precision is yet to be expected in theoretical studies. Nevertheless, such studies can help to elucidate the nature of the physical situation, while this remains but imperfectly understood. Unfortunately, it is from this viewpoint that many theoretical studies merit adverse criticism and, in many situations of interest, realistic theoretical studies must be considered to lag behind experimental ones.

The simplest situation of stress-wave propagation amenable to theoretical investigation is that obtaining under one-dimensional conditions as produced exactly in a semi-infinite medium subjected to a uniform loading over its plane surface; this situation is realized approximately under practical conditions near the axis when the target is a large block of material, but clearly deviations occur wherever edge-effects are significant. In general, there is a non-zero isotropic lateral stress and a corresponding zero strain, and hence the situation differs radically from that for a bar with a finite cross-section. There is an obvious parallel between the present situation and the (classic and simpler) analogous situation in compressible fluid dynamics, first investigated by Riemann.

Some previous theoretical investigations of such one-dimensional anelastic stress-wave propagation in solids that may be mentioned here are those due to E. H. Lee [Quart. Appl. Math. **10** (1953), 335–346; MR **14**, 602] and K. B. Broberg [J. Appl. Mech. **22** (1955), 317–323; Kungl. Fortifikationsförvaltningen Befastningsbyrån Rap. 109–12 (1956)]. The natural and most convenient method of solution of the boundary-value problems that arise is based, at least in part, upon the theory of characteristics of hyperbolic partial differential equations in two independent variables. However, the details of the procedures required—these include integration along characteristics—become rather complicated when the constitutive physical equations adopted are aimed to simulate the more complex aspects of mechanical behaviour, as previously mentioned, implying values of wave-velocities dependent upon the history of the deformation. In addition, complexities may arise due to the building-up of simple waves, causing shock waves, and to interactions between waves and to reflections of waves at free surfaces. In practice, therefore, the changing pattern of stress waves soon becomes quite intricate even with relatively simple types of constitutive physical equations; moreover, even the general features of this pattern may not be too apparent at the outset for any particular case and a high degree of accuracy is normally required in the detailed numerical work involved. In these circumstances, it is not easy to reach general conclusions concerning the features of stress-wave propagation and separate cases are probably still best treated largely on their own merits.

The present paper is a bold attempt at a systematic investigation of one-dimensional stress-wave propagation in a semi-infinite medium subjected to a prescribed smooth loading-unloading pressure pulse at the surface. Initially, the medium is homogeneous, isotropic and stress-free. The constitutive physical equations for the medium

correspond to the following type of mechanical behaviour taken to approximate the true ones. Incremental elastic strains are governed by a generalized Hooke's law wherein the shear modulus μ is assumed constant for elastic deformation and zero for plastic deformation and the bulk modulus $k(p)$ is assumed to increase with increasing hydrostatic pressure p according to a two-parameter formula, derived from quantum mechanics and fitted to P. W. Bridgman's experimental data obtained under reversible isothermal conditions, as proposed by D. C. Pack, W. M. Evans and H. J. James [Proc. Phys. Soc. **60** (1948), 1–8] in order to provide an extrapolation (beyond about 100 kilobars) to high explosive pressure levels (about 300 kilobars), the possibility of phase changes being excluded. Incremental plastic strains, loading and unloading, are governed by a law of constant shear stress so that there is no work-hardening. Thus the mechanical behaviour of the medium is elastic/perfectly plastic, exhibits no effect analogous to the Bauschinger effect and is independent of rate-of-strain. In addition, physical constants are taken to be independent of temperature and all changes of state are supposed to be adiabatic; these changes are reversible (isentropic) only under purely elastic conditions (neglecting hysteresis effects) and are irreversible under both plastic and shock conditions. However, to the contrary in numerical illustrations, the physical data actually used in pressure-density relations are for reversible isothermal changes of state, although the incompatibility with the true physical data should not be serious. The analysis is made in terms of engineering stress and strain (both quantities measured positive in compression) and Lagrangian co-ordinates are employed, wave velocities then being Lagrangian. Therefore, for (compression) loading, the uni-axial stress-strain relation comprises an essentially linear elastic path (up to the Hugoniot elastic limit) followed by a concave-upwards plastic path, so that the elastic wave velocity $\sqrt{\{(k_1+4\mu/3)/\rho_1\}}$ is constant and the plastic wave-velocity $\sqrt{\{k/\rho_1(1-\varepsilon_x)\}}$ (where ρ_1 is initial density and ε_x is axial strain) is increasing with stress. For elastic unloading from a plastic state, the path is taken as linear with the elastic wave velocity (strictly $\sqrt{\{(k+4\mu/3)/\rho_1(1-\varepsilon_x)\}}$) having a constant value that exceeds the elastic wave velocity for loading, but, under the conditions envisaged (see later remarks), the elastic wave velocities are higher than that value of the plastic wave velocity just previously applying. For plastic unloading, the path is still concave upwards but the plastic wave velocity is initially reduced and further reduces with decrease in stress. Clearly, the circumstances imply that plastic loading continuous waves (i.e., simple waves) must ultimately build-up into plastic shock waves (i.e., discontinuous waves) which are propagated at velocities governed by the Rankine-Hugoniot (finite) equations.

The paper continues with a rather general analytical discussion of certain main features and events of interest in the continuous wave system initiated by a smooth loading-unloading pressure pulse. In this pulse (perhaps representative of an impact loading), the pressure increases continuously from zero up to some maximum value, is held constant for a time, and then decreases continuously to zero. It should be remembered that attention is here confined to the case when the elastic wave velocity exceeds the shock wave velocity, and therefore there is a limit imposed on the maximum pressure in the pulse, for a given material. The first complication considered is the primary interaction between the plastic loading wave and the overtaking elastic unloading wave, it being assumed for the moment that the former simple wave does not build-up into a shock wave before this interaction is completed. It is possible for either a reduced plastic loading wave to be propagated forwards or the plastic loading wave to be annulled and an elastic unloading wave to be propagated forwards, but always there is a reflected elastic loading wave. The next complication considered is the secondary interaction between the reflected elastic loading wave and the oncoming plastic unloading wave. The latter wave may be completely annulled (in which case part of the former wave travels back to the free surface, there to be reflected as an elastic unloading wave) or merely reduced, but in either case an elastic unloading wave is propagated forwards. Still further interactions, leading to a final elastic wave pattern, are possible, but these are not discussed analytically.

In the central part of the paper that follows, there is discussion of shock-wave formation and propagation and of the possible associated interactions with the oncoming elastic unloading wave. As already mentioned, due to the enhanced incompressibility of the material at high pressure, the plastic loading stress-strain curve is concave upwards: thus higher stresses are propagated more rapidly than lower ones and the leading part of the stress pulse being propagated becomes progressively steeper and is ultimately vertical. The simple-wave solution breaks down at this vertical front and, exactly as occurs in the corresponding situation of compressible fluid dynamics, the plastic loading front now propagates as a shock wave, i.e., as a very short pulse across which stress, strain and particle velocity change continuously, albeit very rapidly. It is shown that this narrow shock wave is well approximated by a discontinuous shock wave governed by the Rankine-Hugoniot equations. The shock changes are not isentropic but energy is dissipated as the shock wave is propagated through the material and an appreciable temperature rise can occur. The true pressure-density relation (known as a Hugoniot relation) across the shock wave can be determined from experimental data; the difference between the Hugoniot and isentropic relations (taken to coincide at zero pressure) increases with pressure. The author gives an approximate calculation (based upon experimental data for the partition of plastic irreversible work between structural changes and heat dissipation and upon the reversible isothermal pressure-density relation, i.e., Bridgman's experimental data) to show for a particular case (aluminium) that the shock changes are virtually isentropic for moderate shock strengths involving strain jumps up to 0.05, although this result becomes less true at higher shock strengths. Here, it is assumed that the material behind the shock front does continue to satisfy sufficiently well the original isentropic pressure-density relation, and it is on this basis that the building-up of the continuous plastic loading wave into a single (or multiple) shock wave is treated. The discussion now concerns the interaction between the shock wave, formed through the building-up of the plastic loading wave, and the overtaking elastic unloading wave. The interaction proceeds in a manner rather similar to that already discussed for the interaction of simple waves: the shock front is gradually reduced,

completely or only partly (when a reduced elastic unloading wave or shock wave is propagated forwards, respectively); an elastic loading wave is reflected backwards; and now the interaction path coincides with the shock-front path. Discussion is also given of the more involved interaction between the elastic unloading wave and the plastic loading wave when the front is building-up over different sections to form a series of shocks prior to the formation of a single shock.

The paper concludes with a numerical illustration of the interaction shock formation solution. The situation considered is for 24 S-T aluminium alloy and for a hypothetical stress-wave profile which comprises a parabolic loading pulse and an exponential unloading pulse at the beginning of the interaction. The peak stress is 35 kilobars (cf. the yield stress in simple compression of 3.5 kilobars and the Hugoniot elastic limit of 7 kilobars) and the variation of the ratio of the lengths of these pulses provides a means of varying the time to shock formation in comparison with the time of interaction. A full numerical account is given of a suitable case when the continuous plastic loading front first starts to build-up into a shock at some stage during the overtaking by the elastic unloading wave.

This paper is primarily a theoretical study of certain fundamental features of one-dimensional elastic-plastic wave propagation, particularly with respect to shock-wave formation and to interaction between various types of waves. The study is pursued subject to various simplifying assumptions, largely necessary either for mathematical reasons or for lack of physical knowledge and data. In its mathematical detail, this interesting paper is markedly superior to other available published work in this area. The analysis presented demands careful attention for which the reader is rewarded. Here, it has only been possible to summarize briefly the basic features of the work.

The criticism of the paper by the present reviewer is aimed primarily to indicate certain limitations and shortcomings from a physical viewpoint. It would be valuable to develop similar work for pressure pulses representative of chemical high explosive conditions (when the peak pressure exerted by the gaseous explosion products is achieved in a fraction of a micro-second and this pressure thereafter falls rapidly to zero in a few micro-seconds), for plates of finite thickness (in order to study effects—such as spalling—consequent upon shock-wave reflection at the rear surface), and for metals (such as aluminium and lead) when the shock wave velocity can exceed the elastic wave velocity under practical conditions of interest. The paper omits any mention of the thermodynamical derivation of isentropic pressure-density relations from reversible isothermal ones (to which P. W. Bridgman's experimental data are only directly applicable) or of the experimental determination of Hugoniot pressure-density relations; see, for example, K. B. Broberg [loc. cit.] and M. H. Rice, R. G. McQueen and J. M. Walsh [*Solid state physics*, Vol. 6, pp. 1–63, Academic Press, New York, 1958; MR **21** #5351]. Under high-pressure conditions, the correct pressure-density relations need to be used in constructing the constitutive physical equations; as has been seen, the application made in the paper is to pressure levels low in comparison with those occurring under, say, explosion conditions. The marked effects on mechanical behaviour (e.g., initial yielding) of metals induced at high rates-of-strain are not yet sufficiently well understood for their incorporation in a detailed theory of the present kind, but certainly such physical effects must be expected to modify the detailed picture of the propagation and distortion of stress pulses.

H. G. Hopkins (Sevenoaks)

4234:

Karunes, B.; Onat. E. T. Plastic-wave propagation effects in transverse impact of membranes. J. Appl. Mech. **27** (1960), 172–176.

The problem of the dynamic behaviour of a thin metal diaphragm under the action of applied transverse pressure has received much attention [for a general reference, see R. H. Cole, *Underwater explosions*, Princeton University Press, Princeton, N.J., 1948, Ch. 10]. This problem is of interest in relation to studies of structural damage due to explosions. The deformation produced in diaphragms has also been used to assess the relative strengths of underwater explosions (see Cole, loc. cit.).

The true physical problem of the dynamic deformation of a diaphragm is very complicated and considerable simplifications of actual mechanical behaviour are necessary to provide a basis for tractable theoretical investigations. In the present paper, attention is given to the case when the diaphragm has the shape of a long rectangle with its long edges attached to two parallel fixed supports, their distance apart being such that the diaphragm is initially flat and unstressed. The situation envisaged is therefore effectively one-dimensional. The analysis developed involves the following simplifications and assumptions. The diaphragm is thin so that it supports only direct tensile stress (and not direct compressive stress, shear stress or bending stress) and is made of plastic-rigid material. Therefore the diaphragm is represented by a membrane of incompressible material that remains rigid below the yield stress. Work-hardening is linear, with respect to engineering strain, and there is no rate-of-strain dependence. The applied pressure is impulsive and generates initially a uniform transverse velocity to the membrane.

The motion of the membrane involves two types of waves of discontinuity—transverse and longitudinal—that are propagated across the membrane. These waves can undergo reflections. When the membrane comes to rest, generally it is wedge-shaped and non-uniform in thickness.

The mechanics of wave propagation is discussed directly in terms of finite equations, making appeal to the fundamental conservation laws. However, the detailed nature of the wave propagation is dependent upon the value of a certain non-dimensional rate-of-hardening parameter. There are two distinct cases I and III, corresponding to 'high' and 'low' values of this parameter and an intermediate case II corresponding to a single value of this parameter. Moreover, cases I and III are characterized by the occurrence of more than one phase of the motion. Let A, A' and D be the edges and the centre of the membrane. In the first phase of the motion, let B, B' and C, C' be the positions of the inwards-moving transverse and longitudinal wave-fronts. There is always, of course, symmetry about D. The essential features of the wave motion are summarized as follows.

Case I. The first phase of the motion involves the longitudinal wave C moving ahead of the transverse wave

B; AB is extending at a fixed angle to AA'; BD is contracting parallel to AA' while moving transversely at the initial impulsive velocity; and AC has a reduced uniform thickness while CD has the original uniform thickness. The second phase of the motion begins when the two opposing longitudinal waves C and C' meet at D. A reflected longitudinal wave E moves outwards from D towards the incoming transverse wave B, the membrane thinning behind E. The third phase of the motion begins when the transverse wave B meets the longitudinal wave E, and now a transverse wave F preceded by a longitudinal wave G moves outwards but (due to previous work-hardening) only a transverse wave H moves inwards. The portion FH is moving towards D, a situation which induces compressive stresses (see later remarks made on case III). This phase terminates when the transverse wave H reaches D. In certain particular illustrative examples, the numerical results obtained show that at this stage almost all of the kinetic energy imparted to the membrane has been dissipated in plastic work. Therefore the further final stages of motion cannot result in any significant geometrical changes and may accordingly be neglected.

Case II. Here, the motion simply involves a composite longitudinal-transverse wave B–C (B coalescing with C) moving inwards from A towards D. When this composite wave B–C reaches D, the kinetic energy of the membrane has been reduced to zero, and the motion therefore ceases after only one phase.

Case III. The first phase of the motion begins with the transverse wave B moving ahead of the longitudinal wave C; ACB is extending at a fixed angle to AA'; BD is contracting and moving transversely at the initial impulsive velocity; and AC has a reduced uniform thickness while CBD has the original uniform thickness. The second phase of the motion begins when the transverse waves B and B' have reached D. At this instant, CD and $C'D$ have longitudinal velocities directed towards D and this situation causes compressive stresses to occur subsequently in the neighbourhood of D. Strictly, a membrane which has no bending stiffness would at once become unstable under the action of compressive stresses. However, if there is just sufficient bending stiffness to preclude the occurrence of instability but not to affect other features of the problem, then the following second phase of the motion appears possible, assuming that yielding in compression does not occur. The longitudinal wave C continues to move inwards and a stationary transverse front B positioned at D now causes material in CD and CD' converging at D to be deflected along a direction normal to AA' to form an extending 'tail' to the membrane surface, now Y-shaped and not V-shaped as previously. Although this type of continuation of the motion seems possible, it is not discussed in detail and in any event even a small degree of bending stiffness would result in considerable modification of the nature of the plastic flow near D just described.

The authors' numerical results are confined to case I, three examples for different combinations of rate-of-hardening and initial velocity being considered in detail. From these results it is concluded, in particular, that the over-all deflected shape is well approximated by a modification of a formula due to G. I. Taylor and that the reflection of the longitudinal waves C and C' from D is mainly responsible for the pronounced thinning that occurs in the central part of the membrane.

As the authors observe, compressive stress conditions (for both cases I and III) can be set up in the membrane. Such a situation can only be properly catered for through the introduction of bending stiffness. Although it is clear that this would involve some complications it might be possible to undertake this refinement, perhaps mainly at the expense of involved computation.

It should be noted that the present problem of a rectangular membrane is very much simpler than that of a circular membrane [see Cole, loc. cit. and G. E. Hudson, J. Appl. Phys. **22** (1951), 1–11; MR **12**, 654]. However, certain features may be expected to be qualitatively the same in the two cases. The particular interest and merit of the present investigation is that a clear picture of the deformation can be obtained in an extremely simple manner. *H. G. Hopkins* (Sevenoaks)

4235:

Andriankin, È. I.; Koryavov, V. P. Shock waves in a variably compacting plastic medium. Dokl. Akad. Nauk SSSR **128** (1959), 257–260 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 966–969.

The problem of a spherically symmetric explosion in a medium, which, at the wave front, has a compaction depending on the amplitude of the pressure and which is plastic in back of the wave front, is studied. The plastic material is supposed to satisfy the Prandtl plasticity condition and is incompressible. For high pressure the compaction at the wave front is assumed to be constant. For lower pressure, that is, for values of the compaction up to a certain value, a power law between pressure and compaction is assumed. That happens at a certain distance from explosion locus. At the boundary the authors assume that the products of the explosion expand like an ideal gas with an isentropic exponent.

The results are compared with those obtained by A. S. Kompaneec [same Dokl. **109** (1956), 49–52] which assumed a constant compaction along the wave front, and with experimental results obtained by A. N. Romašov, V. N. Rodionov, and A. P. Suhotin [same Dokl **123** (1958), 1283–1286; MR **21** #4636]. In conclusion the authors show that from a distance of the order of 6–7 times the radius of the charge, the compaction must be considered variable because it is important.

N. Cristescu (Bucharest)

4236:

Drucker, D. C. Plasticity. Structural mechanics, pp. 407–455. Pergamon Press, New York, 1960.

This is a survey of some developments in the mechanics (but not physics) of plastic solids between 1948 and 1958. Attention is mostly confined to infinitesimal straining of ideal elastic/plastic or rigid/plastic solids, and especially when these are non-hardening and with identical yield function and plastic potential.

Uniqueness and extremum principles are reviewed at some length, from the standpoint of the author's concept (or postulate) of stability of material. Solutions to some particular boundary-value problems are touched on. The proofs of the theorems of limit analysis for rigid/plastic solids are recapitulated, and a variety of applications mentioned. Effects of changes in geometry, buckling, shakedown, fracture, and dynamic loading, also receive brief discussion. *R. Hill* (Nottingham)

4237:

Hodge, Philip G., Jr. ★Plastic analysis of structures. McGraw-Hill Series in Engineering Sciences. McGraw-Hill Book Co., Inc., New York-Toronto-London, 1959. xiv + 364 pp. $10.50.

This textbook is divided into two parts. Part I deals with the limit analysis of beams, frames and grillages subjected to proportional loading. Chapters are also included on elastic displacements, general (non-proportional) loading, limit design and the influence of axial forces on bending capacity. The presentation is based on the limit theorems of Greenberg and Prager and extensive use is made of the principle of virtual work.

In Part II a unified treatment is developed for structures subject to combined stress, such as beams in bending and torsion, and plates and shells. Most of the analysis is less than five years old and some is presented for the first time. The author has taken care to recast the material from research papers into a fully consistent form. The approach adapted is based on the general theory for the plastic analysis of structures in terms of generalized stresses and strains proposed by W. Prager [Proc. Eighth Internat. Congr. Theoret. Appl. Mech., Istanbul, 1952, vol. II, pp. 65–72, Faculty of Science, Univ. of Istanbul, 1955]. This is combined with the author's technique of approximating the yield surface in a piecewise linear manner in order to simplify the expressions for the internal energy dissipation [P. G. Hodge, Jr., Proc. Colloq. on Deformation and Flow of Solids, Madrid, 1955, pp. 147–170, Springer, Berlin, 1956]. The net result is to bring the theory of structures subject to combined stress within the reach of a much wider group of engineers. The solutions obtained are approximations for the problem with the true yield condition, being upper or lower bounds, depending on whether the piecewise linear yield condition lies outside or inside the true yield surface respectively. Part II also contains a chapter on plane stress, in which the use of stress and velocity discontinuities for finding approximate solutions is illustrated. A final chapter on impact loading deals with the case of a cantilever beam loaded with a tip mass, and also with a circular cylindrical shell loaded by a radial pressure pulse.

In several instances experimental data taken from the literature are quoted as evidence in support of the appropriateness of the analysis for metal structures. Load-deflection graphs are drawn for five types of structure and in each case the theory is in substantial agreement with the test results. The reviewer considers this comparison to be a particularly valuable feature of the book, even when the universal agreement indicated might lead an incautious student to conclude, despite the author's remarks in the Introduction, that these and the other analyses are equally valid under all circumstances. An important instance of the latter occurs in Chapter 3, where three and four-story building frames are analysed on the basis of bending action only. Instability is often a serious problem in frames of this type. The risk of misunderstanding on this point might have been reduced still further by including some test data for cases in which a particular form of analysis proves to be inadequate.

R. M. Haythornthwaite (Ann Arbor, Mich.)

4238:

Safrončik, A. I. Unsteady flow of visco-plastic material in a circular tube. Prikl. Mat. Meh. **24** (1960), 149–153 (Russian); translated as J. Appl. Math. Mech. **24**, 200–207.

This is a one-dimensional analysis of flow of a Bingham plastic through a circular pipe, the flow being produced by a specified time dependent pressure gradient. Previous work on this problem is discussed briefly.

J. L. Ericksen (Baltimore, Md.)

4239:

Astrahan, I. M.; Grigoryan, S. S. On the complete system of equations of a compressible visco-plastic fluid. Prikl. Mat. Meh. **23** (1959), 1142–1143 (Russian); translated as J. Appl. Math. Mech. **23**, 1637–1639.

The constitutive equations here proposed are essentially the same as those introduced by Oldroyd [Proc. Cambridge Philos. Soc. **43** (1947), 100–105; MR 8, 240] to describe Bingham materials. To cover compressible materials, they propose a definite relation between density and pressure, pressure being identified with mean normal stress. The authors propose this as a theory applicable to certain types of soils.

J. L. Ericksen (Baltimore, Md.)

4240:

Ivlev, D. D. On the determination of displacements in the Galin problem. Prikl. Mat. Meh. **23** (1959), 987–988 (Russian); translated as J. Appl. Math. Mech. **23**, 1414–1416.

This brief note contains some footnote remarks to the author's previous paper with the same title in Prikl. Mat. Meh. **21** (1957), 716–718 [MR **19**, 1210].

R. Hill (Nottingham)

4241:

Tarasenko, E. N. Methods of determining limits of flow in bending. Zap. Leningrad. Gorn. Inst. **36** (1958), no. 3, 184–187. (Russian)

4242:

Bandić, I. Sur l'équation différentielle d'un problème de technique étudié par M. R. Gran Olsson. Z. Angew. Math. Mech. **40** (1960), 370.

4243:

McNabb, A. A mathematical treatment of one-dimensional soil consolidation. Quart. Appl. Math. **17** (1959/60), 337–347.

The first part of the paper gives a mathematical treatment of the consolidation of a layer of saturated clay based upon assumptions stated by K. Terzaghi in his book *Erdbaumechanik auf bodenphysikalischer Grundlage* [Deuticke, Leipzig, 1925]. The theory, which is restricted to one dimension and neglects the motion of the clay, leads to a non-linear partial differential equation which, together with the boundary condition appropriate to the standard consolidation test, is then subjected to a dimensional analysis. It is shown that if the hydrological state of the clay depends only upon the void ratio ε, and the pressure P and the permeability K are each functions of ε only, then the total consolidation of a semi-infinite medium at time t is proportional to $t^{1/2}$. This result represents a considerable widening of the conditions under which the "square root law" holds.

There follows a discussion of the experimental determination of the function $P(\varepsilon)$ and $K(\varepsilon)$, and an empirical

approach to the more difficult problem of finding $K(\varepsilon)$ is proposed.

Finally the linearization of the basic partial differential equation is considered. It is shown that the general condition $K=-k(\partial/\partial P)(\varepsilon+\frac{1}{2}\varepsilon^2)$ governs this operation, where k is independent of ε. The author shows how the linear consolidation theories of Terzaghi [loc. cit.] and Taylor and Merchant [J. Math. and Phys. **19** (1940), 167–185] satisfy this condition. Solutions of the linearized problem are then obtained by Laplace transform methods, firstly under conditions corresponding to those imposed in the standard consolidation test, and secondly for consolidation under a uniformly increasing boundary load. The solutions are used to calculate the total consolidation as a function of time, and the role of secondary consolidation is clearly displayed in the results. A number of useful series expansions are derived, but no curves or numerical results are given. *P. Chadwick* (Sheffield)

4244:

Chadwick, P. Thermoelasticity. The dynamical theory. Progress in solid mechanics, Vol. 1, pp. 263–328. North-Holland Publishing Co., Amsterdam, 1960.

As a contribution to a series of survey articles on continuum mechanics the paper summarizes our present knowledge in the field of wave propagation in a perfectly elastic, isotropic solid taking the cross-coupling between temperature field and deformation into account. Discussion is restricted to small deformations. First the basic equations of thermoelasticity are derived using the laws of conservation of mass, momentum and energy and the second law of thermodynamics. Temperature changes are assumed to be small. In order to demonstrate the characteristic behavior of thermoelastic waves plane waves, i.e., waves depending on one space coordinate only are considered. A thorough discussion is given. Then the general three-dimensional boundary value problem of thermo-elastodynamics is stated. A comparison of magnitude of terms in the governing equations based on the properties of metals shows to a very close approximation that, when the deformation is due entirely to heat conduction, dynamic terms may be neglected and the temperature is independent of the deformation.

The remainder of the article is devoted to a review of the few known exact solutions of boundary value problems of thermo-elastodynamics (Rayleigh waves, longitudinal oscillations of an infinite circular cylinder). Problems for future research are indicated.

H. Parkus (Vienna)

4245:

Satoh, Tunezo. An optimal length of the cylinder under the thermal stressed state. Mem. School Sci. Engrg. Waseda Univ. Tokyo. No. 23 (1959), 127–134.

STRUCTURE OF MATTER

See also 4495.

4246:

Skrotskiĭ, G. V.; Kokin, A. A. On the theory of nuclear paramagnetic resonance in liquids. Soviet Physics. JETP **36** (9) (1959), 335–339 (481–487 Ž. Èksper. Teoret. Fiz.)

Authors' summary: "The quantum theory of magnetic resonance absorption due to Kubo and Tomita [J. Phys. Soc. Japan **9** (1954), 888–919] is used to describe the phenomenon of nuclear paramagnetic resonance in liquids. The thermal motion of the molecules, which leads to a narrowing of the absorption line, is taken into account on the basis of diffusion theory. The transverse and the longitudinal relaxation times and the correction to the gyromagnetic ratio are computed." *H. Mori* (Kyoto)

4247:

Kaplan, J. I.; Meiboom, S. Double-quantum transitions in nuclear magnetic resonance spectra of liquids. Phys. Rev. (2) **106** (1957), 499–501.

Authors' summary: "Observations of double-quantum transitions in the nuclear magnetic resonance spectra of liquids are reported. The presence of spin-spin interaction causes the double-quantum lines to form multiplets. A theoretical explanation of the multiplet structure is given."

4248:

Dzyaloshinskiĭ, I. E.; Pitaevskiĭ, L. P. van der Waals forces in an inhomogeneous dielectric. Ž. Èksper. Teoret. Fiz. **36** (1959), 1797–1805 (Russian); translated as Soviet Physics. JETP **9**, 1282–1287.

4249:

Dzyaloshinskiĭ, I. E.; Lifšic, E. M.; Pitaevskiĭ, L. P. van der Waals forces in liquid films. Ž. Èksper. Teoret. Fiz. **37** (1959), 229–241 (Russian); translated as Soviet Physics. JETP **10** (1960), 161–170.

4250:

Dzyub, I. P. An application of the method of Green's functions in solid state theory. Dokl. Akad. Nauk SSSR **130** (1960), 1241–1243 (Russian); translated as Soviet Physics. Dokl. **5**, 125–127.

The method of Green's functions is applied to the exciton state in a solid. By assuming a suitable decoupling of the three-particle Green function, which takes account of the interaction of an excited electron and a hole in the occupied states, the basic equation is derived to determine the retarded two-particle Green function. It is remarked that this equation gives Frenkel's exciton, Mott's exciton and also the collective oscillation of an electron gas under proper conditions. *R. Kubo* (Tokyo)

4251:

Tomášek, M.; Koutecký, J. Theory of Tamm surface states in approximation higher than tight-binding approximation. Czechoslovak J. Phys. **10** (1960), 268–274. (Russian summary)

The authors investigate the conditions for the existence of surface states on crystals in the one electron approximation. It is assumed that the periodic potential is disturbed in the surface layer. Solutions of the Schrödinger equation are obtained in the form of a linear combination of Wannier functions. The dependence of the energy of the bulk states on the component of the wave vector

which is perpendicular to the surface is approximated by the first two terms of a Fourier expansion. Criteria for the existence of surface states are then given in terms of the perturbing surface potential and the two Fourier coefficients in the above-mentioned expansion.

H. Statz (Waltham, Mass.)

4252:

Belov, K. P.; Levitin, R. Z. On the thermodynamical theory of antiferromagnetic transformations. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 3, 129–133. (Russian)

4253:

Döring, W.; Simon, G. Die Richtungsabhängigkeit der Magnetostriktion. Ann. Physik (7) **5** (1960), 373–387.

Authors' summary: "In Einkristallen läßt sich die Magnetostriktion durch einen symmetrischen zweistufigen Tensor beschreiben, dessen Komponenten von der Magnetisierungsrichtung abhängen. Diese Abhängigkeit wird mit Hilfe eines gruppentheoretischen Verfahrens für alle Kristallklassen in eine solche Form gebracht, daß sie der Kristallsymmetrie genügt. Die Resultate sind nicht nur auf die Magnetostriktion, sondern auch z.B. auf den Widerstand in magnetischen Einkristallen und in der Theorie der Gitterschwingungen anwendbar."

4254:

Lax, M.; Mengert, P. Influence of trapping, diffusion and recombination on carrier concentration fluctuations. Phys. and Chem. Solids **14** (1960), 248–267.

Authors' summary: "Carrier concentration fluctuations are found to be relatively large in regions of low concentration. Diffusion is shown to add a $(1/\omega)^{3/2}$ boundary term (important at high frequencies) to the usual volume recombination spectrum. This result is found to be valid independent of the geometry or the number of dimensions. But for the case of surface recombination it is multiplied by a frequency dependent probability that an electron having left the region will not return. Traps of large cross section are characterized by a fast trapping rate and an exceedingly slow effective recombination rate caused by multiple trapping. The fast rate is unaffected by diffusion. The slow rate is shown by a method of low mode isolation to be approximately describable in terms of an effective diffusion constant reduced in about the same ratio as the recombination. The resulting $(1/\omega)^{3/2}$ spectrum is thus shifted to a low frequency region and increased enormously in strength."

4255:

Rosenstock, H. B. Infrared absorption by one-dimensional ionic lattices. Phys. and Chem. Solids **15** (1960), 50–53.

Author's summary: "The absorption spectrum due to the atomic motion in a one-dimensional ionic lattice, considered to consist of point particles of equal mass and alternating charge with Coulomb interaction and nearest-neighbor repulsion, is found to consist of a line of shape $[-\ln(\omega_{max}-\omega)]^{1/2}/(\omega_{max}-\omega)^{3/2}$."

4256:

Koutecký, J.; Tomášek, M. Contribution of the theory of the Shockley surface states. I. General formulation and the case of zero deformation of the potential on the Goodwin-Artmann model. Phys. and Chem. Solids **14** (1960), 241–247.

Authors' summary: "Within the framework of the tight-binding method, general equations have been set up for the calculation of orbital energies and wave functions of the electron in a crystal model in which an interaction between atomic s orbitals and p orbitals is assumed. Simple explicit expressions are derived for the case of zero deformation of the potential of the crystal surface. From a discussion of the hybridization of the wave functions of the volume and surface states the connection with the concept of free valence localized in the crystal surface follows."

4257:

Elliott, R. J.; Loudon, R. Group theory of scattering processes in crystals. Phys. and Chem. Solids **15** (1960), 146–151.

Authors' summary: "A general method is given for determining selection rules for scattering of various types in crystals by using the properties of space groups. The method is particularly useful for those lattices which contain screw axes or glide planes, and a diamond is considered in detail. Examples of particular kinds of selection rules which are of interest in silicon and germanium are examined."

FLUID MECHANICS, ACOUSTICS

See also 4238, 4239, 4356, 4396, 4530.

4258:

Nazarov, G. I. Exact solutions of axisymmetric flows of an ideal fluid. Prikl. Mat. Meh. **23** (1959), 388 (Russian); translated as J. Appl. Math. Mech. **23**, 545–546.

It is shown that the imaginary part of

$$y^{1/2}\int_0^z \frac{dW(\zeta)}{d\zeta}\,H\left(\frac{3}{2},\,-\frac{1}{2},\,1,\,\frac{z-\zeta}{2y}\right)d\zeta$$

satisfies the equation for Stokes's stream function; x, y are coordinates in a meridian plane measured along and perpendicular to the axis of symmetry, $z=x+iy$, the function W is arbitrary, and H is the hypergeometric function.

W. R. Dean (London)

4259:

Vâlcovici, Victor. Sur le mouvement des fluides barotropes. Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **21** (1956), 288–296.

De l'introduction: "Le mouvement permanent des fluides barotropes dénués de viscosité met en évidence une certaine surface dont le plan tangent contient la vitesse et le tourbillon. Cette propriété a été établie déjà par H. Poincaré. La surface a été appelée ultérieurement 'surface de Bernoulli'.

"Dans le présent travail en considérant les surfaces de Bernoulli dans le cas général nous arrivons à la conclusion

que l'étude du mouvement du fluide se réduit proprement dit au mouvement en deux dimensions sur les surfaces rigides de Bernoulli et que ce mouvement est déterminé par deux fonctions, φ et ψ, dont la première fournit la vitesse du fluide par ses dérivées partielles par rapport aux coordonnées curvilignes, q^1 et q^2, sur la surface de Bernoulli, et la seconde, les lignes de courant, qui sont données par l'équation $\psi=\text{const.}$ De cette manière le mouvement permanent du fluide barotrope en trois dimensions se présente comme étant réduit au mouvement en deux dimensions, réduction analogue à celle que l'on connait pour les mouvements irrotationnels des fluides incompressibles. L'analogie s'étend aussi aux équations que les fonctions φ et ψ doivent satisfaire, et qui sont des équations du type elliptique, généralisant l'équation de Laplace. Dans certains cas particuliers, qui d'ailleurs comprennent le cas spécial du mouvement plan mentionné ci-dessus, la combinaison $\varphi+i\psi$ apparait comme une fonction analytique d'un certain argument complexe."

4260:

Kapur, J. N. On axially symmetric superposable flows. Bull. Calcutta Math. Soc. **51** (1959), 25–33.

4261:

Vallander, S. V. On the application of the method of singularities to the calculation of fluid flow in radial-axial turbines. Soviet Physics. Dokl. **123** (3) (1958), 1113–1116 (413–416 Dokl. Akad. Nauk SSSR).

This paper is based upon an earlier one by the same author [same Dokl. **84** (1952), 673–676]. Singularities consisting of source-vortices are introduced at the lattice points with polar coordinates $\xi=\xi_0$, $\theta=\theta_0+kT$ ($k=0, \pm 1, \pm 2, \cdots$; $T=2\pi/n$), where n is the number of blades. A potential function is introduced for the source components of the singularities and a stream function for their vortex components, but the differential equations for both functions are essentially in the same form. These equations are non-homogeneous partial differential equations of the second order involving the Dirac delta-function and the periodic Dirac delta-function with period T. Their solutions are expressed as Fourier series.

M. Marden (Madison, Wis.)

4262:

Chartier, Charles; Bigaud, André; Bureste, Henri. Sur l'écoulement hydrodynamique à l'aval d'un cylindre. C. R. Acad. Sci. Paris **251** (1960), 507–508.

4263:

Kamimoto, Goro; Ishida, Shuji; Akamatsu, Taruaki. An approximate theory of airfoil cascade in subsonic flow. Bull. JSME **3** (1960), 177–246.

Authors' summary: "This paper deals with an approximate theory of airfoil cascades in subsonic flow. The theory was developed by the so-called 'interference method' and Prandtl-Glauert's approximate theory of airfoil. The calculated results of some airfoil cascades coincide well with the ones calculated by the 'hodograph method'. The inverse problem is also discussed in this paper."

4264:

Yen, K. T. On the thrust hypothesis for the jet flap including jet-mixing effects. J. Aerospace Sci. **27** (1960), 607–614.

The "thickness" and "camber" problems for a jet-flapped wing are formally set out, and it is shown that the thrust is $J_\infty+Q_0U_\infty$, where J_∞ is the integral of momentum excess across a contour far behind the wing, and Q_0 is the mass flow coming from within the wing. This result was proved rigorously earlier by Woods [J. Fluid Mech. **1** (1956), 54–60]. The author also notes that in the limit of infinite jet velocity and zero thickness with J finite, so that Q becomes zero, it is possible to show that for an inviscid flow J remains constant along the jet, and the thrust is then independent of the angle of exit. This result too has been known for some years, and in particular formed the basis of the present reviewer's treatment of the problem by thin aerofoil theory [Proc. Roy. Soc. London Ser. A **238** (1956), 46–68; MR **18**, 529]. This represents the "camber" problem in the present discussion; an attack on the thickness problem by the same methods would be interesting. The author discusses the way these results are modified by mixing, closely following the discussion of Stratford [Aero. Quart. **7** (1956), 85–105]. No one would disagree with his conclusions that "the thrust is sensitive to a change in jet angle" and that "the mixing phenomenon plays a very important role in the thrust developed by a jet-flap system", but some qualitative indication of these effects would have been useful.

D. A. Spence (Pasadena, Calif.)

4265:

Kapur, J. N. Transverse component of velocity in a plane symmetrical forced jet of a compressible fluid. Bull. Calcutta Math. Soc. **51** (1959), 34–38.

4266:

Grigoryan, S. S. An approximate solution of the problem of separated flow past an axially symmetric body. Prikl. Mat. Meh. **23** (1959), 951–953 (Russian); translated as J. Appl. Math. Mech. **23**, 1351–1355.

The unsteady motion of a thin axially symmetric body through a compressible gas is considered. It is assumed that a cavity is formed to the rear of the body and that the cavitation number is small. The author is able to find a complete solution for the shape of the cavity in simple terms. Some applications and limitations are discussed.

K. Stewartson (Durham)

4267:

Dumitrescu, D.; Ionescu, V. Comportement des lames déversantes au voisinage du point de détachement. Com. Acad. R. P. Romîne **10** (1960), 187–194. (Romanian. Russian and French summaries)

Authors' summary: "Dans ce travail on étudie le problème du mouvement des fluides pesants à surface libre sous la forme de lames déversantes, au voisinage du point de détachement. Le problème se ramène à la détermination de la fonction de Lévi-Cività $\omega(Z)$ dans le demi-plan supérieur du plan auxiliaire $Z=X+iY$, par des conditions aux limites.

"On cherche la solution du problème comme limite d'une série d'approximations successives $\{\omega_n(Z)\}$ dont la

première représente le mouvement du fluide parfait, non pesant.

"On établit la forme générale des fonctions $\omega_n(Z)$. Dans la dernière partie du travail on donne des indications sur la convergence et l'unicité de la solution."

4268:

Dumitrescu, D.; Ionescu, V.; Crăciun, C. Contribution à l'étude du mouvement des fluides pesants par-dessus des déversoirs en mince paroi. Com. Acad. R. P. Romîne **10** (1960), 195–203. (Romanian. Russian and French summaries)

Authors' summary: "Les résultats théoriques de l'étude du mouvement des fluides pesants à lames déversantes, obtenus dans un travail antérieur [voir l'analyse précédente], ont été appliqués au cas concret du déversoir en mince paroi. Les auteurs ont établi les formules déterminant la forme des surfaces libres, en diverses approximations. Elles ont été appliquées au mouvement par-dessus un déversoir à paramètres arbitrairement choisis.

"Ils ont effectué les calculs numériques pour les quatre premières approximations du mouvement. Le travail ne comprend que les trois premières approximations, parce que la quatrième n'apporte pas de corrections sensibles à la troisième."

4269:

Dumitrescu, D.; Iancu, C. Domaine de validité des courbes caractéristiques des déversoirs en mince paroi. Com. Acad. R. P. Romîne **10** (1960), 205–210. (Romanian. Russian and French summaries)

Authors' summary: "Les essais effectués sur les divers modèles de déversoirs en mince paroi ont permis de calculer l'expression de la fonction $\varphi(\mathcal{F}_4)$, déduite de l'application du théorème π à l'écoulement par-dessus des déversoirs.

"Le nouveau mode d'exprimer la formule du débit a conduit à une expression analytique du coefficient de débit $m=\sqrt{(\frac{1}{2}\mathcal{F}_4)}$, vérifiée expérimentalement. La généralisation de la formule représente une extension par rapport aux formules existant déjà dans la littérature.

"L'article comprend, en outre, des formules pratiques, pouvant être appliquées au laboratoire, pour la mesure du débit évacué par un déversoir en mince paroi."

4270:

Dumitrescu, D.; Ionescu, V. L'étude du mouvement des fluides pesants déversants dans un plan vertical. Com. Acad. R. P. Romîne **10** (1960), 181–186. (Romanian. Russian and French summaries)

Authors' summary: "Les auteurs étudient le mouvement dans un plan vertical, d'un fluide pesant comportant des lames déversantes qui tombent librement dans le champ de la gravitation, y compris le point à l'infini aval de la lame déversante.

"Ils donnent la solution du problème du mouvement sous la forme de la limite d'une suite de fonctions $\omega_n(Z)$, définies dans le demi-cercle de Lévi-Cività. Les fonctions $\omega_n(Z)$ sont conçues comme différentes approximations de la solution du problème.

"A la fin du travail sont mentionnées les conclusions concernant le comportement des approximations à l'infini aval de la lame déversante et les directions dans lesquelles devraient être dirigées les recherches."

4271:

Sutherland, C. D.; Cohen, Hirsh. Finite cavity cascade flow. Proceedings of the Third U.S. National Congress of Applied Mechanics, Brown University, Providence, R.I., June 11–14, 1958, pp. 837–846. American Society of Mechanical Engineers, New York, 1958. xxvii+864 pp. $20.00.

The problem considered in this paper is the steady cavity flow of incompressible inviscid fluid through a cascade, i.e., an infinite array of identical hydrofoils with cavities fully developed on the suction side of the lifting surfaces. The lifting foils are taken to be thin and to be held at a small angle of attack to the approaching flow, but the stagger angle, chord-to-foil-spacing ratio (or the solidity) and the cavity length are left arbitrary. The analysis is carried out with application to the linearized theory of Tulin, and the problem solved for any stagger angle and solidity in terms of the cavitation number, exit angle and exit velocity. The effect of changing solidity on cavitation number, exit angle and lift coefficient is obtained. The relation between the lift coefficient and cavitation number is also found for a wide range of the solidity ratio; there is apparently a radical change in this relation as the stagger angle changes from 0 to 60 degrees.

This problem has important applications to the theory of cavity flows in rotating machinery such as compressors, turbines and perhaps propellers. The authors have made their significant contribution by extending the cavity flows through cascades to the case of finite cavities. It may be noted, however, that in linearization the present authors choose to approximate the location of the planar cavity boundaries to be parallel to the flow direction at upstream infinity. In a similar problem of cascade flows with infinitely long cavities considered by Acosta [unpublished], the cavity boundaries are approximated to lie along the direction of the camber of the foils. The difference between the results due to the different approximations of the cavity boundaries in these two cases is not readily available. *T. Yao-tsu Wu* (Pasadena, Calif.)

4272:

Tasai, Fukuzō. On the damping force and added mass of ships heaving and pitching. Rep. Res. Inst. Appl. Mech. Kyushu Univ. **7** (1959), 131–152.

The exact linear theory for horizontal cylinders is used. The cross-sections treated here are Lewis forms, i.e., they are derived from the circle by a conformal transformation with two adjustable coefficients. The results of computations are presented for a variety of cross-sections and are compared with earlier approximate methods due to Grim. Grim's high-frequency method for damping is generalized to all Lewis forms, and it is found that actually the resulting integral gives a very good idea of the damping over a wide range of frequencies.

The two-dimensional values of damping and virtual mass are used to obtain, by a strip-theory, values for the damping and virtual mass of two ship-models used in experiments by Golovato and by Gerritsma. The experimental coefficients of damping in heave agree less well with

this strip theory than the experimental coefficients of damping in pitch and of virtual mass.

F. Ursell (Cambridge, England)

4273:

Nye, J. F. The response of glaciers and ice-sheets to seasonal and climatic changes. Proc. Roy. Soc. London. Ser. A **256** (1960), 559–584.

The basic theory of "kinematic waves" [M. J. Lighthill and G. B. Whitham, same Proc. **229** (1955), 281–316, 317–345; MR **17**, 309, 310] is applied, with appropriate developments, to several interesting problems of glacier flow. In the one-dimensional flow of a glacier or icesheet, conservation of ice requires that the flow rate $q(x, t)$ and thickness $h(x, t)$ are related by $\partial q/\partial x + \partial h/\partial t = a$, where $a(x, t)$ is the given rate of accumulation (positive for snowfall and negative for ablation). It is assumed further that q is a known function of h, x and possibly $\partial h/\partial x$. Suitable empirical formulas with $q \propto h^n$, $n \approx 3$, are used in calculations; the kinematic wave speed $\partial q/\partial h$ is then nu where $u = q/h$ is the flow velocity of the glacier.

If the accumulation rate a is changed suddenly by a small amount, the response is small in regions of extension where $du/dx > 0$ and unexpectedly large in compression regions where $du/dx < 0$. In fact the thickness increases exponentially in compression regions until relieved by kinematic waves coming from the extension regions. Glaciers are usually in extension above the snow line (note that $a > 0$ implies $dq/dx > 0$ in the steady state) and in compression below. Thus the theory indicates the observed sensitivity of the lower regions of glaciers to changes in climate.

The second main discussion concerns the response of glaciers to periodic changes in climate, with prediction of the phase lag in the consequent changes in thickness. Estimates are made for both seasonal and long-period changes in climate for the icesheets of Antarctica and Greenland as well as for glaciers.

Brief accounts of methods for predicting the advance and retreat of the snout of a glacier, and methods for treating variations of valley width, etc., are included.

G. B. Whitham (Cambridge, Mass.)

4274:

Miles, John W. On the generation of surface waves by turbulent shear flows. J. Fluid Mech. **7** (1960), 469–478.

This paper is concerned with the generation of waves by wind under the combined influence of (i) the random surface pressure fluctuations arising from the atmospheric turbulence and of (ii) the surface pressure distribution (proportional to the surface slope) associated with the interaction between the waves and the wind. It is assumed that these are independent, and that the resultant surface pressure is the sum of (i) and (ii). It is found that for a certain time (independent of the magnitude of the turbulent pressure spectrum), the energy in a Fourier component grows linearly, and then approaches an exponential rate of growth, characteristic of an instability theory. The wind duration, or time at which the rate of growth increases, is of order 10^2 wave periods for the components travelling much slower than the wind and increases rapidly as the phase velocity of the wave approaches that of the wind.

The reviewer believes this to be an important contribution in elucidating the conditions under which the instability theory or the resonance theory describe most accurately the rate of energy transfer from wind to waves.

O. M. Phillips (Baltimore, Md.)

4275:

Finkelstein, A. B. The initial value problem for transient water waves. Comm. Pure Appl. Math. **10** (1957), 511–522.

The author investigates the general three (space) dimensional motion of a heavy incompressible liquid occupying for time $t \geq 0$ the region $-h \leq y \leq 0$, $-\infty < x, z < \infty$. Present are finite obstacles whose infinitesimal motions (near the rest position of equilibrium) are prescribed. At infinity, the displacement and velocity of all particles are permitted to become unbounded like $O(\exp\{\pi/2h - \varepsilon\}\sigma)$ for any t, where $\sigma^2 = x^2 + z^2$ and $\varepsilon > 0$.

First the author constructs and studies the behavior of the time dependent Green's function $G(x, y, z, \xi, \eta, \zeta; t, \tau)$ which satisfies $\Delta G = \delta(x-\xi)\delta(y-\eta)\delta(z-\zeta)$ throughout the region and homogeneous boundary conditions. Using G the problem of finding the velocity potential is reduced to solving an integral equation. The uniqueness of the solution is proved by means of energy integrals.

W. Littman (Minneapolis, Minn.)

4276:

Longuet-Higgins, M. S. Mass transport in the boundary layer at a free oscillating surface. J. Fluid Mech. **8** (1960), 293–306.

In a previous paper [Philos. Trans. Roy. Soc. London. Ser. A **245** (1953), 535–581; MR **15**, 362], the author predicted that just below the free surface boundary layer of a gravity wave in a real fluid, the mass-transport gradient should be exactly twice the value given by Stokes irrotational theory. This is a consequence of the inwards diffusion of a second order vorticity field from the free surface. The present paper describes careful experiments which confirm the earlier theoretical result.

Reviewer believes that this prediction and its experimental verification constitute one of the more important contributions in recent years to gravity wave theory. One implication is that potential solutions involving higher order effects should be treated with caution; in particular, expressions for mean wave momentum and mass transport velocity based on irrotational solutions are not relevant to a real fluid.

O. M. Phillips (Baltimore, Md.)

4277:

Takano, K. Effets d'un obstacle parallélépipédique sur la propagation de la houle. Houille Blanche **15** (1960), 247–267. (English summary)

Author's summary: "This article is devoted to a theoretical treatment, by means of linear approximations, of two analytically similar but physically distinct phenomena. Consider an open wave flume with a horizontal bottom. A rectangular obstacle S, the top and bottom of which are parallel, and whose width is the same as that of the flume, can disturb two-dimensional wave propagation in two ways: (I) S is partially immersed in the liquid. The incident waves are partially reflected by the upstream face of S, pass between the lower face of S and the bottom of the flume under pressure, and reappear downstream as transmitted waves. (II) S is completely

submerged. The incident waves pass over the sill formed by S and set up a periodic disturbance, which may be very complex, over and downstream of S. It is thus seen that the work had to be done in two phases and the author has presented these parts separately for the reader's convenience."

The field of flow is in each case the sum of three regions of constant depth. The form of expansion for such regions is known, and the coefficients must be determined so that there are no discontinuities. Infinite sets of linear equations are written down, and an approximate solution is found by retaining only a small number of unknowns. The agreement with certain experiments is better than was obtained from earlier and cruder theories.

F. Ursell (Cambridge, England)

4278:

Sagomonyan, A. Ya. Falling of a plane plate on the surface of a compressible fluid. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 2, 49–53. (Russian)

4279:

Popov, N. N. Non-stationary one-dimensional motion of a gas with introduction of heat in a channel of variable cross-section. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 3, 19–27. (Russian)

4280:

Gohin, M. Étude des spectres de houle à proximité des côtes. Houille Blanche **15** (1960), 113–121. (English summary)

This paper is a rather incomplete review of work on wind generated gravity waves when they are interpreted as a stationary random Gaussian process as published elsewhere by other authors. The discussions at the end of the paper are interesting as they reveal a lack of understanding of the full usefulness and the actual limitations of the techniques discussed in the paper.

W. J. Pierson, Jr. (New York)

4281:

Murota, Akira. On the secondary flow in open channels. Tech. Rep. Osaka Univ. **10** (1960), 85–96.

Author's summary: "Relations between the secondary flow in open channels and the acceleration of non-uniform flow are investigated in part I. It is verified analytically and experimentally that a sort of secondary flow can exist in the positive, or negative, accelerated flow (for instance, convergent or divergent channel flows).

"The secondary flow near the branching point of the channel with a distributary and its effect on the distribution of sediment loads transported from the upstream are mentioned in part II."

R. C. MacCamy (Pittsburgh, Pa.)

4282:

Illingworth, C. R. A note on fluctuating heat transfer at small Péclet numbers. J. Fluid Mech. **7** (1960), 442–448.

This note gives the solution for the temperature field when a warm circular cylinder or a warm sphere is held at rest in a fluctuating stream. The heat-transfer equation is simplified in the manner of Oseen's theory for low Reynolds number flow. With the further assumptions of constant fluid properties, corresponding to small temperature differences, and small Mach numbers, the equation can be solved without knowledge of the velocity field. Analytical and numerical results are presented for the mean and fluctuating parts of the heat-transfer coefficient and the ratio of the fluctuating part to its quasi-steady value.

D. W. Dunn (Ottawa, Ont.)

4283:

Morton, B. R. Weak thermal vortex rings. J. Fluid Mech. **9** (1960), 107–118.

From the author's summary: "A similarity solution is obtained to first order in an effective Rayleigh number for very weak thermal vortex rings produced by heat released at a point in a fluid. The vortex propagates slowly in relation to its rate of growth, and the process is dominated by viscous and thermal diffusion."

W. V. R. Malkus (Woods Hole, Mass.)

4284:

Rozenbaum, R. B.; Todes, O. M. Theoretical analysis of the motion of a sphere in a cylindrical tube (with a viscous fluid). Zap. Leningrad. Gorn. Inst. **36** (1958), no. 3, 16–27. (Russian)

4285:

Wilkinson, J. The steady incompressible boundary-layer flow past a flat plate with a parabolic leading edge. Quart. J. Mech. Appl. Math. **13** (1960), 199–209.

Kaplun [Z. Angew. Math. Physik **5** (1954), 111–135; MR **15**, 907] has shown, using parabolic coordinates, that the conventional solution, of the boundary layer equations for flow past a flat plate, may be rendered uniformly valid except at the leading edge, in the limit of vanishing viscosity. The purpose of the present paper is to generalise this result to include flat plates with parabolic leading edges. A degenerate form of paraboloidal coordinates is used and on assuming that the velocity components are effectively functions of one variable only a satisfactorily consistent solution is found, in the limit of vanishing viscosity. Some comments are added on the relation between this solution and Moore's theory [NACA Tech. Note No. 2279 (1951); MR **12**, 872].

K. Stewartson (Durham)

4286:

Merk, H. J. Mass transfer in laminar boundary layers calculated by means of a perturbation method. Appl. Sci. Res. A **8** (1959), 237–260.

Author's summary: "The macroscopic equations for combined heat and mass transfer discussed in a foregoing paper [Appl. Sci. Res. A **8** (1959), 73–99; MR **22** #1332] are simplified and applied to transfer of momentum, heat and mass in laminar boundary layers. The mass transfer is described in a fairly general manner, as was indicated first by Spalding [Proc. Roy. Soc. London Ser. A **221** (1954), 78–99, 100–104; MR **15**, 908]. On the assumption that the Lewis number is unity, the heat transfer is easily related to the mass transfer. This relation is restricted to low values of the Mach number, but it does account for the dependence of the enthalpy on the

concentration, the latter effect having been apparently neglected in the literature hitherto. For flat plates the dependence of the Sherwood number on the Schmidt number and on the differences in concentration is investigated, the latter effect being expressed in terms of a dimensionless transfer parameter B. For low values of B the Sherwood number is obtained as an explicit function of the Schmidt number in the range $0.5 < Sc < \infty$. Furthermore, the Sherwood number is calculated for large values of the Schmidt number and finite values of B. The results of the calculations are compared with some other calculations found in the literature. It is shown that the approximations used in the present paper result in suitable formulae for rapid calculation of the Sherwood number." *R. C. DiPrima* (Troy, N.Y.)

4287:

Rumyancev, V. V. The stability of Maclaurin ellipsoids of a rotating fluid. Prikl. Mat. Meh. **23** (1959), 494–504 (Russian); translated as J. Appl. Math. Mech. **23**, 701–715.

Ten ordinary differential equations for ten functions of time are formulated which describe the classical problem of the motion of a rotating ellipsoid of incompressible inviscid fluid whose particles are attracted to each other by Newton's law and whose surface is at constant pressure. The author investigates stability in the sense of Liapunov of the Maclaurin ellipsoids subject to the restriction that the free surface remains an ellipsoid. Stability is proved for a figure which always remains an ellipsoid of revolution, and for the bifurcated ellipsoid initially in uniform rotation. *L. A. Segel* (Troy, N.Y.)

4288:

Tatsumi, T.; Gotoh, K. The stability of free boundary layers between two uniform streams. J. Fluid Mech. **7** (1960), 433–441.

The authors investigate the stability of free boundary layers by means of the method previously used by Tatsumi and Kakutani for the jet [same J. **4** (1958), 261–275; MR **20** #567] in which the eigenfunction is expanded as a power series in αR. The situation at low Reynolds numbers is found to be universal for all velocity profiles of this type. Curves of constant amplification are calculated as far as $O(R^2)$. The asymptotic form of the neutral curves for $R \to 0$ is $\alpha = R/4\sqrt{3}$, so that the critical Reynolds numbers of these flows are identically zero. The phase velocity is also zero for all disturbances, up to the second approximation. *D. W. Dunn* (Ottawa, Ont.)

4289:

Ellison, T. H. A note on the velocity profile and longitudinal mixing in a broad open channel. J. Fluid Mech. **8** (1960), 33–40.

Dimensional arguments are presented in discussing the velocity profile near the free surface of turbulent flow in a wide open channel. The profile is shown to be of the form

$$u(z) = u_{\max} - 2m^{-1}u_*(1 - z/h)^{1/2},$$

when $z/h \ll 1$, where u_* is the friction velocity at the bottom (when $z = h$) and m is a constant analogous to the von Karman constant for boundary flows. The quantity m could be estimated by precise profile measurements, but observations such as those of Elder [same J. **5** (1959), 544–560; MR **21** #3185] on the longitudinal diffusion in this flow provide a more sensitive method. On the basis of Elder's results, author estimates m to be of the order 0.8. *O. M. Phillips* (Baltimore, Md.)

4290:

Burger, A. P.; van der Lingen, T. W. The damping effect of distributed and concentrated resistances on small perturbations in a uniform flow. Quart. J. Mech. Appl. Math. **13** (1960), 40–48.

Authors' summary: "The damping of two dimensional disturbances by a distributed resistance in a uniform incompressible flow is explored. Parallel plates are chosen to produce the resistance. A quadratic friction law is assumed. When the downstream length of the plates is very short compared to the disturbance wave length, the results approximate previous experimental and theoretical studies using gauze wire screens. Formulae are presented for the damping effect of plates of arbitrary length."

W. V. R. Malkus (Woods Hole, Mass.)

4291:

Dolaptschiew, Bl. Bemerkungen über die Stabilitätsuntersuchungen der Wirbelstrassen. Deutsch. Akad. Wiss. Berlin Schr. Forschungsinst. Math. **4** (1957), 28 pp.

This is an address at the Institute of Applied Mathematics of Humboldt University and the Berlin Academy of Sciences, giving a complete review on the present status of the research on vortex streets. Starting from the early investigations of Mallock and Bénard, through Kármán's investigations up to those in the recent times, the author arrives at the conclusion that the well-known Kármán stability criterion refers to a point stability, whereas it seems that in reality in vortex streets there appears a "space stability" (Prandtl's conjecture). The author describes briefly the following topics in the field in question: (I) stability and instability of vortex streets of various configurations from the standpoint of various stability criteria; (II) properties of vortex streets; (III) inclined motion of the vortex streets; (IV) stabilizing the vortex streets. In more detail the author discusses the following items: degrees of freedom of disturbances, Kármán criterion, law of periodic disturbances, special group disturbances, group disturbances of the first order, and of higher order, generalized condition of stability, criticism of the stability criteria, generalized stability criterion of Kotschin, annihilation of vortex streets, vortex streets near obstacles, vortex streets in compressible media, cylindrical vortex streets and the action of viscosity. *M. Z. v. Krzywoblocki* (E. Lansing, Mich.)

4292:

Case, K. M. Taylor instability of an inverted atmosphere. Phys. Fluids **3** (1960), 366–368.

The author considers a two-dimensional, spatially periodic disturbance of a fluid that has the unperturbed density $A \exp(\beta y)$ above a rigid plane $y = 0$. He shows that the vertical velocity grows like

$$t^{-3/2} \exp\{[4gk^2\beta/(\beta^2 + 4k^2)]^{1/2}t\},$$

where g is the acceleration of gravity and k the wave

number. No reference is made to the earlier work of Eliassen, Høiland and Riis [Institute for Weather and Climate Research, the Norwegian Academy of Sciences and Letters, Publ. No. 1 (1953); MR **16**, 642], who obtained a similar result for $\beta \ll k$.

J. W. Miles (Los Angeles, Calif.)

4293:

Yih, Chia-Shun. A transformation for non-homentropic flows, with an application to large-amplitude motion in the atmosphere. J. Fluid Mech. **9** (1960), 68–80.

Neglecting body forces and dissipative processes the equations for steady flow of a perfect gas can be written

$$u_j \frac{\partial u_i}{\partial x_j} = -\frac{e^{S/C_v}}{p^{1/\gamma}} \frac{\partial p}{\partial x_i}, \quad \frac{\partial u_j}{\partial x_j} + \frac{u_j}{\gamma p} \frac{\partial p}{\partial x_j} = 0, \quad u_j \frac{\partial S}{\partial x_j} = 0,$$

where S is the entropy, and the fundamental result of the paper is that by taking new variables $u_i' = u_i e^{-S/2C_v}$ we eliminate S from the first two equations. Hence to any non-homentropic steady flow corresponds a homentropic one with the same streamlines. An application, involving a similar transformation when body forces are present, is made to lee waves, under restrictions which permit linear approximations. *T. M. Cherry* (Melbourne)

4294:

Gispert, Hans-Günter. Berechnung der Unterschallströmung um ein symmetrisches Joukowski-Profil mit Hilfe eines Variationsproblems. Wiss. Z. Martin-Luther-Univ. Halle-Wittenberg. Math.-Nat. Reihe **6** (1956/57), 803–806.

The paper deals with plane, irrotational, steady state flow of an inviscid but compressible fluid. For subsonic flow around a given profile Bateman's integral over a certain power of the sound velocity, the integration to be made over the exterior of the profile, becomes stationary. The integral goes with linear boundary conditions. In a former paper [same Z. **6** (1956/57), 209–221; MR **19**, 798] the author used the integral in order to find the flow around cylinders of elliptic and especially circular cross-sections. In the present paper the method is extended to noncirculatory flow around a symmetric Joukowski-profile, more precisely the profile obtained by mapping the unit circle $|\zeta| = 1$ of the ζ-plane by means of

$$(z+2)/(z-2\lambda) = (\zeta+1)^2/(\zeta-\lambda)^2.$$

As for Bateman's integral, it is transformed into an integral over $|\zeta| \leq 1$. The integration requires particular attention in the neighborhood of $\zeta = -1$. A numerical example is presented. *H. Bückner* (Madison, Wis.)

4295:

Lyubimov, G. A. On compression of a gaseous cylinder in flow. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1958**, no. 6, 13–17. (Russian)

4296:

Wood, William W.; Parker, F. R. Structure of a centered rarefaction wave in a relaxing gas. Phys. Fluids **1** (1958), 230–241.

The authors study the structure of a centered rarefaction wave in a relaxing gas with zero viscosity. Analytic and numerical methods are used to show that there is a transition from the "frozen" flow at short times to equilibrium flow at long times.

A. H. Taub (Urbana, Ill.)

4297:

Mishra, R. S. On flows behind a three dimensional unsteady curved shock wave. Indian J. Math. **2**, 19–28 (1960).

The author formulates the equations of motion of a perfect fluid satisfying an arbitrary equation of state with an unsteady shock present in a particular non-cartesian coordinate system. The coordinate system used involves the Gaussian coordinates of a point on the shock surface, and the distance along a stream-line emanating from that point. Such a coordinate system was introduced by Taub [Ann. of Math. (2) **62** (1955), 300–325; MR **17**, 208] for the discussion of two-dimensional stationary and pseudo-stationary flows. It was also used by Kanwal [Proc. Amer. Math. Soc. **9** (1958), 201–207; MR **20** #597] to discuss such flows in three dimensions. The present paper extends Kanwal's results to unsteady flows.

A. H. Taub (Urbana, Ill.)

4298:

Borg, S. F. A similarity solution for a blast in free space. J. Franklin Inst. **268** (1959), 446–452.

4299:

Wood, W. W.; Salsburg, Z. W. Analysis of steady-state supported one-dimensional detonations and shocks. Phys. Fluids **3** (1960), 549–566.

Possible steady one-dimensional flows and their stability in a medium in which an arbitrary number of chemical reactions proceed behind an initiating shock are investigated with neglect of transport effects due to viscosity, thermal conduction, diffusion and radiation. For detonations there are stable solutions resulting in an equilibrium final state for detonation velocities equal to or greater than the equilibrium Chapman-Jouguet (C-J) value. It is suggested that these solutions correspond to piston-supported detonations and the equilibrium C-J detonation is stable with respect to removal of the piston support at sufficiently late times. The "normal frozen C-J condition" results in an unstable solution. Pathological detonations in which the region of steady flow terminates at a point of incomplete reaction are identified but not further characterized. *H. L. Frisch* (Murray Hill, N.J.)

4300:

Friedman, M. P. An improved perturbation theory for shock waves propagating through non-uniform regions. J. Fluid Mech. **8** (1960), 193–209.

The linearized treatment of the flow behind a shock propagating through a slightly non-uniform medium breaks down when the shock strength is such that the flow behind is nearly sonic. However the formulas deduced for the perturbations to the shock strength and shock velocity do not exhibit any singular behavior. The author explains why the shock formulas are still correct and shows how to improve the perturbation theory to give a complete approximate solution for the flow behind the shock. The sound waves propagating back from the

shock move with velocity $u-a$, where u is the fluid velocity and a is sound speed. In the singular case this is nearly zero, the waves are nearly stationary and it turns out that the disturbance carried by these waves increases so long as they remain nearly stationary. In the true non-linear theory, the increase in disturbance automatically changes $u-a$ away from zero, the waves move away and the increase of disturbance ceases. Now the linear theory makes no allowance for this effect and always approximates $u-a$ by the value in the undisturbed flow; thus linear theory predicts unbounded disturbances. When this crucial non-linear effect is retained in the mathematical theory the disturbance remains finite for all time. The theory is developed for the specific problem of propagation down a tube of non-uniform cross-section.

A most interesting feature of the improved solution is the formation of secondary shocks in the flow behind the main shock. When the flow behind the shock is well away from sonic, subsidiary shocks, if any, would form far upstream or downstream and would be outside the region of interest. However, near sonic flow subsidiary shocks form soon after the main shock enters the non-uniform region. *G. B. Whitham* (Cambridge, Mass.)

4301:

Cabannes, Henri. Sur l'attachement des ondes de choc dans les écoulements à deux dimensions. C. R. Acad. Sci. Paris **250** (1960), 1968–1970.

The case considered is one of those treated in an earlier paper of the author [Rech. Aéro. No. 71 (1959), 3–9; MR **21** #6186]; viz., fluid velocity and magnetic field are parallel upstream of the shock front. The limiting conditions for shock attachment are determined here; the maximum wedge angle θ_m in a given gas depends on two dimensionless parameters characterizing the upstream flow; i.e., the Mach number x and the ratio, ε^2, of magnetic pressure to γ times static pressure, where γ is the specific-heat ratio. For $\theta>\theta_m$ the shock, if one exists, must be detached. For $\varepsilon^2<1$ the curve of θ_m against x has three branches; for $\varepsilon^2>1$ the diagrams of this note show two of these branches connecting. Families of these curves for various values of ε^2 have been computed for $\gamma=7/5$ and are plotted here.

{For $\varepsilon^2<1$ the three branches intersect the $\theta=0$ axis at values which are recognized as the boundaries of hyperbolic-flow regimes for this type of flow; i.e., the regions of attached waves are just the regions where standing magnetosonic waves exist. It is interesting to note that for $\varepsilon^2>1$ this is not so, for there are regions where the attached shock waves do not reduce to magnetosonic waves at $\theta=0$; they are a "strong" family.}

W. R. Sears (Ithaca, N.Y.)

4302:

Bagdoev, A. G. Propagation of pressure in inhomogeneous fluids. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 6, 43–50. (Russian)

4303:

Powell, Alan. Aerodynamic noise and the plane boundary. J. Acoust. Soc. Amer. **32** (1960), 982–990.

An exposition of the use of reflection arguments in discussing sound generation by turbulence near boundaries. Though rigour is claimed, it can be found lacking in some respects, particularly in the distinction between the "noise generation region" and the region of "stationary and uniform" fluid outside. The principal new conclusion is that the reflection idea is valid for rigid plane boundaries even at non-negligible Mach numbers less than unity.

O. M. Phillips (Baltimore, Md.)

4304:

Simonin, Raymond F. Sur une onde moléculaire sphérique stationnaire. C. R. Acad. Sci. Paris **250** (1960), 4274–4276.

Author's summary: "On montre que les molécules superficielles d'une veine en régime d'écoulement uniforme, percutant un plan d'eau, sont transférées à la surface des bulles d'air, au cours de la genèse hydrodynamique de ces bulles. Le mouvement orbiculaire de transfert est une onde sphérique stationnaire. Les calculs font exclusivement appel aux équations classiques de cinématique. Les résultats obtenus coïncident totalement avec ceux obtenus antérieurement et utilisant la théorie moléculaire de la tension superficielle, exposée par Laplace dans son traité de mécanique céleste."

4305:

Molmud, Paul. Expansion of a rarefied gas into a vacuum. Phys. Fluids **3** (1960), 362–366.

Two solutions of this problem are compared with each other: (i) The free molecular flow approximation. The solution is obtained from a related heat flow problem. (ii) A second solution is obtained from numerical integration of the equations of gas dynamics. No mentioning is made of the method used.

Both solutions (i) and (ii) show the same general behavior, but the reviewer disagrees that there is "very little" difference (50%). *H. Oser* (Washington, D.C.)

4306:

Tolstoy, I.; May, J. A numerical solution for the problem of long-range sound propagation in continuously stratified media, with applications to the deep ocean. J. Acoust. Soc. Amer. **32** (1960), 655–660.

The propagation of low-frequency-sound waves in the deep ocean is treated. The ocean is divided in n layers in each of which the square of the refraction index is assumed to be a linear function in the vertical coordinate: $n_i^2=a_iz+b_i$. Homogeneity in the horizontal direction is assumed. The discontinuities in the first derivative of n^2 at each of the interfaces are believed to be of no influence at the low frequencies considered here.

The wave equation is solved with the aid of Lanczos' τ-method [see C. Lanczos, *Applied analysis*, Prentice Hall, Englewood Cliffs, N.J., 1956; MR **18**, 823], using an IBM 650 Electronic Computer. An error analysis shows that approximately 200 modes suffice to compute the acoustical pressure field with an error between .05 and 1% for distance R (source-receiver) less than 100 km. Ways are indicated to obtain results of higher accuracy as well as for longer distances, the present results being limited mainly by the computer but not by the method.

The results are being presented in 3 graphs.

H. Oser (Washington, D.C.)

4307:

Papadopoulos, V. M. A line source on an interface between two media. J. Fluid Mech. **8** (1960), 41–48.

The method of dynamical similarity [Craggs, Proc. Roy. Soc. London. Ser. A **237** (1956), 372–382; MR **20** #599] is applied to the problem named. The solution of the hydrodynamic problem is somewhat artificial, as it requires the source to supply two different fluids in a definite ratio, but the corresponding electro-magnetic solutions are of more interest.

J. W. Craggs (Newcastle-upon-Tyne)

4308:

Gottlieb, Peter. Sound source near a velocity discontinuity. J. Acoust. Soc. Amer. **32** (1960), 1117–1122.

Author's summary: "The far-field solution for a line and a point source near a tangential velocity discontinuity has been calculated by summing (integrating) the plane waves that make up the source. The exact field integrals were evaluated approximately by the stationary-phase method, and this approximation gives the far field. It was found that the sound was strongly peaked in some directions, and considerably reduced in others. This angular dependence is shown graphically for certain cases. The physical significance of these results is discussed for both subsonic and supersonic motions, and the relationship to the jet-noise problem is suggested."

R. N. Goss (San Diego, Calif.)

4309:

Cytovič, V. N. On the influence of resonance properties of a plasma on the propagation of electro-acoustic waves. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 2, 135–141. (Russian)

4310:

Vol'pert, È. G. The calculation of the elasticity of a gas in a state of vibration. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 6, 37–42. (Russian)

4311:

Lyubimov, G. A. Investigation of stationary surfaces of jump discontinuity in the conductivity of a gas in an electromagnetic field. Izv. Akad. Nauk SSSR. Otd. Tehn Nauk. Meh. Mašinostr. **1959**, no. 5, 9–15. (Russian)

4312:

Zabusky, N. J. Hydromagnetic stability of a streaming cylindrical incompressible plasma. Phys. Fluids **3** (1960), 278–288.

A cylindrically symmetric plasma is in a steady state of motion, the plasma being incompressible and of zero resistance. This motion is perturbed. The plasma is surrounded by a compressible medium which is non-conducting. The boundary conditions between the plasma and its surrounding medium are that the normal magnetic flux, the normal velocity and the stress are all continuous. The zeros of the dispersion relation are found through a sequence of mappings between three complex planes. The presence of flow introduces overstable modes. If the perturbed quantities do not depend on orientation about the axis of the cylinder, the time-divergencies are removed by flow. If there is orientation dependence, the divergencies are enhanced by flow.

The mathematical notation is complicated and is very inadequately explained by the author.

G. C. McVittie (Urbana, Ill.)

4313:

Kruskal, M. D.; Johnson, J. L.; Gottlieb, M. B.; Goldman, L. M. Hydromagnetic instability in a stellarator. Phys. Fluids **1** (1958), 421–429.

In this paper the authors discuss the hydromagnetic instabilities that occur in stellarators. A thin column of plasma is confined by means of the magnetic field due to a high-current discharge along the column. It was noted earlier [M. D. Kruskal and J. L. Tuck, Proc. Roy. Soc. London. Ser. A **245** (1958), 222–237; MR **20** #626] that when there is imposed an external longitudinal magnetic field whose magnitude is much larger than the field of the discharge current, one should expect instabilities in the form of a lateral displacement of the plasma column into a helix of large pitch. This problem is examined under the conditions which might occur in stellarators during ohmic heating. The stellarator is treated as if it were straightened out to form a right circular cylinder, and its finite length is taken into account by imposing a periodicity condition, the period being the length of the stellarator. It is shown that the external conductors have a negligible effect on the instability. There exists a critical current below which the system is stable for displacements which vary as $f(r)e^{i(m\theta+kz)}$, where r, θ, z are the cylindrical co-ordinates. Further if, instead of a purely surface current, there exist longitudinal volume currents, instabilities exist for values of m greater than one. For $m=1$, there is instability for currents greater than a critical value, whose magnitude depends on the geometry of the system, the plasma cross-section, the magnitude of the longitudinal confining magnetic field and on the direction of the current. The experimental results on the stellarators $B-1$, $B-2$, $B-3$ are in agreement with these results for $m=1$. There is no experimental indication of modes greater than one.

F. C. Auluck (Delhi)

4314:

Halbwachs, F.; Hillion, P.; Vigier, J.-P. Internal motions of relativistic fluid masses. Nuovo Cimento (10) **15** (1960), 209–232. (Italian summary)

A fluid motion confined to a world tube in space-time is said to be characterized by a symmetric stress-energy tensor $t_{\mu\nu}$ and a current vector j_μ, each of which is divergence-free. "Global" quantities are defined in terms of averages and moments of these quantities taken over a space-like plane three-dimensional section of the tube. The equations satisfied by these global quantities as a consequence of the conservation theorems are given.

The internal motions are assumed to be describable by a variational principle whose Lagrangian depends on global quantities alone. A particular Lagrangian is discussed in detail in terms of the relativistic analogues of the Euler angles. The Hamiltonian formulation is also given.

A. H. Taub (Urbana, Ill.)

4315:

Coburn, N. Intrinsic form of the characteristic relations for a perfect compressible fluid in general relativity and

non-steady Newtonian mechanics. J. Math. Mech. **9** (1960), 421–437.

The author expresses the four-velocity vector field of a fluid in terms of a space-like vector n^i of the normal cone and a time-like vector t^i of the bicharacteristic cone. The conservation equations satisfied by an isentropic, perfect, non-self-gravitating fluid in general relativity are then expressed in terms of the sound speed a, the speed of light c, and derivatives of sound speed along the directions of n^i and t^i and the derivatives of these two vectors. The case when the characteristic surfaces form a family of hyperplanes and an auxiliary vector field generates a family of straight lines is discussed in some detail and compared to the Newtonian approximation.

The stress energy tensor used in the first part of the paper to describe the fluid differs from that proposed by Eckart and used by the reviewer. In the last section of the paper the author shows that if the latter stress energy tensor is used and certain modifications are made then the characteristic relation derived in an earlier paper [same J. **7** (1958), 449–481; MR **20** #5623] and used in this paper remains valid. *A. H. Taub* (Urbana, Ill.)

4316:

Gerber, Charles-Sébastien. Sur l'existence du suintement dans la filtration à surface libre à travers un massif perméable à faces verticales. C. R. Acad. Sci. Paris **250** (1960), 3776–3778.

Author's summary: "La recherche directe de la solution analytique dans le plan du potentiel complexe montre l'existence théorique de la ligne de suintement quel que soit le niveau du plan d'eau à l'aval du massif."

4317:

Cerf, Roger. Sur la variation du coefficient de diffusion de translation avec la viscosité du solvant: terme correctif à la loi d'Einstein. C. R. Acad. Sci. Paris **250** (1960), 3599–3601.

Author's summary: "En complétant l'équation de Langevin par une force dérivant d'un potentiel périodique on trouve une expression du coefficient de diffusion de translation qui diffère de celle d'Einstein lorsque la viscosité du solvant est faible."

4318:

Gordeev, G. V. Fluidization (with suspended particles). Zap. Leningrad. Gorn. Inst. **36** (1958), no. 3, 65–73. (Russian)

4319:

Škadov, R. I. On the movement of suspended particles in a gas flow. Zap. Leningrad. Gorn. Inst. **36** (1958), no. 3, 139–145. (Russian)

OPTICS, ELECTROMAGNETIC THEORY, CIRCUITS

See also A3647, 4312, 4313, 4512.

4320:

Straneo, Paolo. Teoria delle dimensioni fisiche. I. Archimede **12** (1960), 1–9.

4321:

Herzberger, Max. ★Modern geometrical optics. Pure and Applied Mathematics. Vol. 8. Interscience Publishers, Inc., New York; Interscience Publishers Ltd., London; 1958. xii+504 pp. $15.00.

This book is an excellent exposition of the author's many important contributions to geometrical optics in the period following the publication of his previous book *Strahlenoptik* [J. Springer, Berlin, 1931]. It presents not only a consistent theoretical system for geometric optics, but it also constitutes an invaluable account of modern mathematical techniques for the practical optical designer. The aim of the book is nothing less than to develop a mathematical model of an optical system that is complex enough so that all of the characteristics of the geometrical optical image can be obtained from it, and at the same time is simple enough to be useful to the practical designer.

Part I (66 pages) is a development of formulas for tracing rays through an optical system. The formulas are particularly suitable for electronic computers. Part II (76 pages), on Gaussian optics, is a development of the first order approximation theory by a method (due to the author) using Gaussian brackets. This leads to an investigation of the effect on the constructional elements of an optical system of variation of system data. The laws of optical image formation, including the development of the Hamilton characteristic function (or eikonal) of a system, is the subject of part III (42 pages). A complete mathematical treatment of concentric systems is given in part IV (20 pages). As the author states: "If it were possible to extend the methods of this chapter to systems with rotational symmetry, the problems of lens design would change from an art to a science." Part V (36 pages) is a treatment of optical systems with rotational symmetry. Part VI (44 pages) is an exposition of the contributions of Allvar Gullstrand to modern geometrical optics. These results are then used in part VII (82 pages) to develop the author's image error theory. Finally, in part VIII (36 pages), we have the author's mathematical model for an optical system. One remaining short section (6 pages) indicates how the methods can be applied in the case of inhomogeneous media.

Some of the mathematical tools used in this book are worthy of special mention. The treatment is simplified by considerations of symmetry. The author's concept of a diapoint (the intersection of the image ray with the plane that passes through the object point and the axis of symmetry) also results in a simplification. The treatment of the chromatic aberrations is facilitated by using dispersion formulae that are linear functions of the indices. *G. L. Walker* (Providence, R.I.)

4322:

Drude, Paul. ★The theory of optics. Translated from the German by C. Riborg Mann and Robert A. Millikan. Dover Publications, Inc., New York, 1959. xxi+546 pp. $2.45.

This is a reprint of the 1902 translation of Drude's *Lehrbuch der Optik* [S. Hirzel, Leipzig, 1900]. When the book was written it was a thorough and rather complete advanced textbook on optical theory. Particular attention was devoted to the subjects most prominent in optical research at that time—the optical properties of matter

and applications of thermodynamics to optics—both fields where Drude made substantial contributions.

The book is in three parts. Part I, Geometrical optics, has chapters on: The fundamental laws, Geometrical theory of optical images, Physical conditions for image formation, Apertures and the effects depending upon them, and Optical instruments. Part II on Physical optics consists of two sections. The first section on the General properties of light has chapters on: The velocity of light, Interference of light, Huygens' principle, Diffraction of light, and Polarization. The second section on Physical optics is devoted to Optical properties of bodies and has chapters on: Theory of light, Transparent isotropic media, Optical properties of transparent crystals, Absorbing media, Dispersion, Optically active substances, Magnetically active substances, and Bodies in motion. Part III is concerned with Radiation and has chapters on: Energy of radiation, Application of the second law of thermodynamics to pure temperature radiation, and Incandescent vapors and gases.

G. L. Walker (Providence, R.I.)

4323:

Zaborov, V. P. Isometric transformation of constant-thickness lenses. Radiotehn. i Elektron. **4** (1959), 584–591 (Russian); translated as Radio Engrg. and Electronics 4, no. 4, 36–47.

Author's summary: "A family of lenses is considered in this paper which are obtained by isometric transformation of a constant-thickness lens with variable index of refraction. The bodies of the equivalent lenses have axes of rotation normal to the plane of the lens profile and indices of refraction which are a function of a single radius. The phase-front at the output is plane. Metal-air analogues to the lens family are found. Their mean surfaces are surfaces of revolution."

4324:

von Schelling, H. Die Geometrie des beidäugigen Sehens. Optik **17** (1960), 345–364. (English and French summaries)

Author's summary: "According to R. K. Luneburg the visual space is of a constant, non-vanishing Riemannian curvature. Under the assumption that the visual space is defined by a distance function which is invariant to affine transformations of physical Cartesian coordinates, it is proved: (1) that the sign of the curvature is negative, as expected by Luneburg; and (2) that completely determined, purely mathematical rules exist for mapping the momentary visual space into the physical space. The mapping procedure deviates from methods discussed by Luneburg and his followers. They depend on an unknown function of a single variable which has to be found by the application of psychological tests.

"For the first time the mapping is solved explicitly for the three-dimensional case. The visual images of physical perpendicular lines are computed. Various two-dimensional examples are summarized graphically.

"In the last section it is mentioned that the method can be generalized to four dimensions, represented by Minkowski's coordinates $(x, y, z; ict)$. A theory of the combined perception of space and time would be an analogue to the special theory of relativity."

4325:

Četaev, N. G. The problem of Klein. Prikl. Mat. Meh. **24** (1960), 23–32 (Russian); translated as J. Appl. Math. Mech. **24**, 27–39.

In this posthumous paper (dated 1941) "the further development of Hamilton's analogy between mechanics and optics" is called Klein's problem. The author presents "the solution" by throwing into the Lorentz form the bilinear invariant of a linear Hamiltonian system, this system being such as to describe small variations from a stable motion. *T. M. Cherry* (Melbourne)

4326:

Laikin, Milton. Automatic computation of dielectric films. J. Opt. Soc. Amer. **50** (1960), 721–722.

Author's summary: "A method is presented whereby the transmission and reflection from multiple-layer dielectrics may be calculated. The equations are put into a form readily adapted to large computing machinery. A seven-layer film combination is computed for various wavelengths."

4327:

Longuet-Higgins, M. S. Reflection and refraction at a random moving surface. I. Pattern and paths of specular points. J. Opt. Soc. Amer. **50** (1960), 838–844.

Author's summary: "Light falling from a point source on a ruffled surface produces a pattern of images, which move about over the surface. The image points correspond to the maxima, minima, and saddle points of a certain function. It is shown that the images are generally created in pairs, a maximum with a saddle point or a minimum with a saddle point, and that the total numbers of maxima, minima, and saddle points satisfy the relation

$$N_{ms}+N_{mt} = N_{ss}+1.$$

"The process of creation or annihilation of images is studied in detail, and also the tracks of the image points, in certain special cases. It is shown that closed tracks may be common. This is confirmed by photography of the sea surface." *W. J. Pierson, Jr.* (New York)

4328:

Longuet-Higgins, M. S. Reflection and refraction at a random moving surface. II. Number of specular points in a Gaussian surface. J. Opt. Soc. Amer. **50** (1960), 845–850.

Author's summary: "The number of specular points reflected in a random Gaussian surface is determined theoretically under the following alternative conditions: (1) when the surface is perfectly long crested (two-dimensional); (2) when the surface is three-dimensional but isotropic; (3) for quite general surfaces, provided that the observer and the source of radiation are both at a great distance from the surface.

"The results can be applied to the similar problem when the surface forms the boundary of two refracting media." *W. J. Pierson, Jr.* (New York)

4329:

Longuet-Higgins, M. S. Reflection and refraction at a random moving surface. III. Frequency of twinkling in

a Gaussian surface. J. Opt. Soc. Amer. **50** (1960), 851–856.

Author's summary: "When light is reflected or refracted at a moving Gaussian surface, the observer sees a number of moving images of the source, which appear or disappear generally in pairs; such an event is called a 'twinkle'. In the present paper the number of twinkles per unit time is evaluated in terms of the frequency spectrum of the surface and the distance of the source O and observer Q, on the assumption that the surface is Gaussian and that OQ is perpendicular to the mean surface level.

"A solution is found first for a single system of long-crested (or two-dimensional) waves, and then extended to the case of two such systems intersecting at right angles.

"The rate of twinkling is found to depend, apart from a scale factor, on two parameters of the surface, one of which, α, increases steadily with the distance of O or Q from the surface; the other, d, discriminates between waves of standing type and waves of progressive type. Over a considerable range of α, the rate of twinkling is almost independent of d, but for large values of α the rate is much greater for standing waves than for progressive waves; waves of intermediate type are included in the analysis." *W. J. Pierson, Jr.* (New York)

4330:

Marchand, Nicole; Faget, Jean; Fert, Charles. Diffraction de Fresnel par le bord d'un écran semi-transparent et déphasant. C. R. Acad. Sci. Paris **250** (1960), 4135–4137.

Authors' summary: "Les auteurs étendent au cas d'un écran semi-transparent et déphasant la construction géométrique qui permet, à partir de la spirale de Cornu, un tracé rapide de la figure de diffraction de Fresnel par le bord d'un écran et une discussion commode des phénomènes."

4331:

Rehmann, G. Koordinatentransformation und Huygenssches Prinzip. Optik **17** (1960), 383–391. (English and French summaries)

Author's summary: "It is shown that the dynamic treatment of physical reference systems requires, in general, a dynamic transformation of which the kinematic Lorentz transformation is simply a special case. Huygens' principle proves to be an anticipation of the dynamic transformation in the field of geometrical optics. General conclusions are drawn from the close analogy which emerges between mechanics and optics."

4332:

Henry, Irvin G. Phase oscillations in high current synchrotrons. J. Appl. Phys. **31** (1960), 1338–1342.

Author's summary: "The theory of phase oscillations in the synchrotron, first given by Bohm and Foldy, is extended to cover the case where the amplitude of the accelerating voltage is not constant and where the accelerated bunch is itself sufficiently large to produce an appreciable part of the accelerating voltage. It is found that the phase oscillation is more stable when the driving frequency is less than the natural frequency of the accelerating electrode system, and that the plate characteristic of the driving amplifier has a negligible effect on the phase stability."

4333:

Igrickiĭ, A. L. Calculation of electron trajectories in TWT periodic focusing structures. Radiotehn. i Élektron. **5** (1960), 255–263 (Russian); translated as Radio Engrg. and Electronics **5**, no. 2, 112–124.

Author's summary: "A fairly exact and general method is proposed for calculating electron trajectories in TWT focusing systems with periodic magnetic field. This method enables one to determine the configuration of the electron beam in the tube under arbitrary conditions of entry of electrons into the periodic field."

4334:

Vlasov, A. D. On Brillouin electron flows and the boundary-particle concept. Radiotehn. i Élektron. **5** (1960), 264–268 (Russian); translated as Radio Engrg. and Electronics **5**, no. 2, 125–130.

Author's summary: "The stability of electron beams calculated on the basis of the assumption of the presence of boundary particles and laminar motion of electrons is considered. It is shown that this widely used procedure does not always give valid results, leading in a number of cases to beams of unstable structure."

4335:

Filippov, Yu. F. Electron beam motion with periodic variation of velocity in channels in a lossy medium. Radiotehn. i Élektron. **4** (1959), 233–240 (Russian); translated as Radio Engrg. and Electronics **4**, no. 2, 107–116.

Author's summary: "The phenomena arising in the motion of an electron beam with periodically varying velocity through a porous medium with ohmic losses are examined theoretically.

"It is shown that in distinction to the case of electron beam propagation in vacuum, exponentially growing waves exist for all values of the beam and medium parameters. It is also shown that the presence of thermal velocities does not lead to the appearance of critical frequencies."

4336:

Troickiĭ, Yu. V. Effect of a magnetic field on thermal spreading of a beam in an electron gun. Ž. Tehn. Fiz. **30** (1960), 25–30 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 22–27.

Author's summary: "We consider an axially symmetric electron gun with a magnetic field whose lines of force coincide with the electron trajectories. It is shown that if the magnetic field is strong, it is possible to suppress spreading of electron beam due to the thermal velocities of the electrons.

"As an example, we consider a Pierce gun with a conical beam. An electron gun with a magnetic field can be used in conjunction with a magnetic focusing system in microwave tubes."

4337:

Rosen, Philip. Scattering of electromagnetic waves by longitudinal plasma waves. Phys. Fluids **3** (1960), 416–417.

Since a longitudinal wave in a plasma produces a periodic variation in its electron density, a plasma wave may be represented as a medium having a periodic, relative dielectric constant. The reflection of electromagnetic waves from such a medium is considered, and it is shown that the electric vector satisfies the partial differential equation which is characteristic of Bragg reflection. The Bragg angle is calculated from a dispersion relation and is shown to depend on the wavelength of the electromagnetic wave, the plasma frequency, and the Debye length. The formula is proposed as a possible basis for the measurement of either the electron density or the temperature of the plasma.

R. D. Kodis (Providence, R.I.)

4338:

Bogdanov, E. V.; Kislov, V. Ya.; Černov, Z. S. Interaction between electron stream and plasma. Radiotehn. i Èlektron. **5** (1960), 229–238 (Russian); translated as Radio Engrg. and Electronics **5**, no. 2, 75–89.

Authors' summary: "The problem of interaction of a bounded electron stream with plasma is considered. The dispersion equation is obtained and the conditions for an increasing h.f.-signal are analysed. A system in which a modulated electron stream interacts with a gas-discharge plasma located in a longitudinal magnetic field is experimentally investigated. In the range from 30 to 3 cm. plasma amplification up to 40 dB was obtained."

4339:

Livšic, M. S. A mathematical problem connected with the theory of longitudinally polarized particles. Dokl. Akad. Nauk SSSR **131** (1960), 797–800 (Russian); translated as Soviet Physics. Dokl. **5**, 341–344.

4340:

Rostoker, Norman; Rosenbluth, M. N. Test particles in a completely ionized plasma. Phys. Fluids **3** (1960), 1–14.

Authors' summary: "Starting from the Liouville equation, a chain of equations is obtained by integrating out the coordinates of all but one, two, etc., particles. One 'test' particle is singled out initially. All other 'field' particles are assumed to be initially in thermal equilibrium. In the absence of external fields, the chain of equations is solved by expanding in terms of the parameter $g=1/nLD^3$. For the time evolution of the distribution function of the test particle, an equation is obtained whose asymptotic form is of the usual Fokker-Planck type. It is characterized by a frictional-drag force that decelerates the particle, and a fluctuation tensor that produces acceleration and diffusion in velocity space. The expressions for these quantities contain contributions from Coulomb collisions and the emission and absorption of plasma waves. By consideration of a Maxwell distribution of test particles, the total plasma-wave emission is determined. It is related to Landau's damping by Kirchhoff's law. When there is a constant external magnetic field, the problem is characterized by the parameter g, and also the parameter $\lambda=\omega_c/\omega_p$. The calculation is made by expanding in terms of g, but all orders of λ are retained. To the lowest order in g, the frictional drag and fluctuation tensor are slowly varying functions of λ.

"When $\lambda\ll 1$, the modification of the collisional-drag force due to the magnetic field, is negligible. There is a significant change in the properties of plasma waves of wavelength greater than the Larmor radius which modifies the force due to plasma-wave emission. When $\lambda\gg 1$, the force due to plasma-wave emission disappears. The collisional force is altered to the extent that the maximum impact parameter is sometimes the Larmor radius instead of the Debye length, or something in between. In the case of a slow ion moving perpendicular to the field, the collisional force is of a qualitatively different form. In addition to the drag force antiparallel to the velocity of the particle, there is a collisional force antiparallel to the Lorentz force. The force arises because the particle and its shield cloud are spiralling about field lines. The force on the particle is equal and opposite to the centripetal force acting on the 'shield cloud'. It is much smaller than the Lorentz force."

W. P. Allis (Cambridge, Mass.)

4341:

Elmore, William C.; Tuck, James L.; Watson, Kenneth M. On the inertial-electrostatic confinement of a plasma. Phys. Fluids **2** (1959), 239–246.

4342:

Klein, Milton M.; Brueckner, Keith A. Plasma propulsion by a rapidly varying magnetic field. J. Appl. Phys. **31** (1960), 1437–1448.

Authors' summary: "An investigation has been made of the motion of a plasma under the action of a rapidly varying magnetic field from a stationary coil. The analysis has been done for two cases: (1) the plasma is assumed completely diamagnetic so that the ionization history of the plasma may be ignored and (2) the nondiamagnetic case where the ionization process is considered in detail. For the diamagnetic case the analysis yields a coupled set of nonlinear ordinary differential equations which have been put in similarity form and solved for a wide range of the similarity parameters. An illustrative example is given to show how to use the results to obtain detailed information for a given physical system."

4343:

Noerdlinger, Peter D. Stability of uniform plasmas with respect to longitudinal oscillations. Phys. Rev. (2) **118** (1960), 879–885.

This paper relates the dispersion formula for longitudinal oscillations in an infinite, uniform, collision-free plasma with no magnetic field to the complex potential of a line charge distribution on the real axis of the phase velocity plane. While a similar procedure had been worked out by L. R. Walker [J. Appl. Phys. **25** (1954), 131–132; MR **15**, 488], the present work goes into greater detail. Aside from showing that single-peaked distributions are stable, it shows that those with sharp (nondifferentiable) minima are not. Information is obtained on wavelengths and rates of growth of growing oscillations. The examples of two interpenetrating hot plasmas and of

counter streaming electrons and ions are discussed. A generalization of the method to purely transverse waves is introduced. *H. A. Haus* (Cambridge, Mass.)

4344:

Kellogg, Paul J.; Liemohn, Harold. Instability of contra-streaming plasmas. Phys. Fluids **3** (1960), 40–44.

Previous work by Kahn [Rev. Mod. Phys. **30** (1958), 1069–1071] and Parker [Phys. Rev. **112** (1958), 1429–1435] for the case of zero temperature is generalized by considering perturbations on Maxwellian distributions associated with the two streams. For equilibrium density distributions it is found that instabilities are prevented, provided that the stream temperatures are sufficiently high. Curves specifying the conditions of instability and the boundary between the stable and unstable regions are drawn. Applications to solar particle streams are discussed. *K. C. Westfold* (Sydney)

4345:

Filimonov, G. F. Growing waves propagating in a plasma. Radiotehn. i Élektron. **4** (1959), 75–87 (Russian); translated as Radio Engrg. and Electronics **4**, no. 1, 122–140.

Author's summary: "A unique solution of the one-dimensional linearized kinetic Vlasov equation describing the propagation of high-frequency signals generated by an external signal is found. Electron beams with a square Maxwellian electron velocity distribution in the undisturbed state are examined as examples. It is shown that for small electron-gas temperatures the usual single-velocity approximation can be used. Analysis of the expressions obtained makes it possible to establish the propagation direction of the natural waves in the plasma and to solve the problem of growing waves in a rectilinear electron flow."

4346:

Sitenko, A. G.; Kiročkin, Yu. A. Excitation of waves in a plasma. Ž. Tehn. Fiz. **29** (1959), 801–807 (Russian); translated as Soviet Physics. Tech. Phys. **4** (1960), 723–729.

The dispersion equation for electromagnetic waves in a plasma in a magnetic field is found, but there are errors in the formulae which come from neglecting the electron mass too soon. The radiation of waves into the plasma by surface and line currents perpendicular to the magnetic field is also given. *W. P. Allis* (Cambridge, Mass.)

4347:

Denisov, N. G. The absorption of radio waves in resonance regions of an inhomogeneous plasma. Radiotehn. i Elektron. **4** (1959), 388–397 (Russian); translated as Radio Engrg. and Electronics **4**, no. 3, 51–65.

Author's summary: "With reference to the problem of the propagation of electromagnetic waves in an inhomogeneous magneto-active plasma, we investigate the passage of waves through a region where the refractive index becomes infinite. In this region the frequency of the external field coincides with the natural frequency of plasma oscillations. A calculation of the effect of absorption of the extraordinary wave in the resonance region is carried out neglecting the effect of interaction. The absorption of the ordinary and extraordinary waves is also investigated taking into account their interaction. The results obtained are discussed with reference to the case of wave propagation in the ionosphere."

4348:

Gordeev, G. V. The influence of end boundaries upon the rotation of plasma in a magnetic field. Ž. Tehn. Fiz. **29** (1959), 759–762 (Russian); translated as Soviet Physics. Tech. Phys. **4**, 683–686.

The rotation of an infinitely long plasma contained between coaxial cylinders, carrying a radial current, and located in an axial magnetic field was considered in a previous paper by Gordeev and Gubanov [same Ž. **28** (1958), 2046–2054]. In this paper end plates are added. Solution is by expansion in Fourier and Bessel series. *W. P. Allis* (Cambridge, Mass.)

4349:

Skuridin, G. A.; Stanyukovič, K. P. Approximate solution of the problem of motion of a conducting plasma. Dokl. Akad. Nauk SSSR **130** (1960), 1248–1251 (Russian); translated as Soviet Physics. Dokl. **5**, 132–136.

The authors give a formal asymptotic solution of the non-linear system of partial differential equations which describe the one-dimensional motion (along the x-axis) of a non-viscous, finitely conducting plasma in which the magnetic field is perpendicular to the velocity throughout. The key device which yields an asymptotic solution (for large ω, the frequency of periodic solutions) is taken from the theory of linear hyperbolic partial differential equations, wherein it is known that the limit of harmonic solutions for ω approaching infinity is the same as the propagation of the discontinuity (or discontinuous solution) of the time dependent hyperbolic partial differential equation.

In the present paper the authors use the assumption $\phi = A(x, y, z, t) \exp\{i\omega f(x, y, z, t)\}$ to solve the non-linear equation $\phi_t + u\phi_x = \kappa\phi_{xx}$, wherein u is the single component of the velocity and ϕ is related to the single component H of the magnetic field by $H = \phi_x$. By using the fact that ω is large and by imposing supplementary conditions the authors obtain expressions for u and ϕ. With these two functions the other functions, pressure and density, in the non-linear system are determined to within boundary conditions which may be imposed to treat a shock wave. *M. Kline* (New York)

4350:

Chandrasekhar, S.; Kaufman, A. N.; Watson, K. M. Properties of an ionized gas of low density in a magnetic field. III. Ann. Physics **2** (1957), 435–470.

[For part II see Brueckner and Watson, Phys. Rev. **102** (1956), 19–27.]

Authors' summary: "Equations are derived which describe the hydrodynamic properties of an ionized gas in a strong magnetic field and in states close to an initial stationary state. The development is based on the Boltzmann equation in which the effects of collisions between the constituents of the gas are ignored. The theory is, therefore, applicable only for following the evolution of the gas for durations which are short compared to the thermalization time. The method of solution followed is

essentially one of expansion in inverse powers of the strength of the impressed magnetic field; and in the first approximation the Boltzmann equation is reducible to a one-dimensional inhomogeneous wave equation which describes the motions of the particles along the magnetic field lines." *M. L. Goldberger* (Princeton, N.J.)

4351:

Chandrasekhar, S.; Kaufman, A. N.; Watson, K. M. Properties of an ionized gas of low density in a magnetic field. IV. Ann. Physics **5** (1958), 1–25.

Authors' summary: "A systematic method is described for solving the Boltzmann equation for the steady states of an ionized gas of low density in a strong magnetic field. On the assumption that the term representing the Lorentz force in the Boltzmann equation dominates all others, the solution is developed as a series in inverse powers of the gyration frequency ω. The solution is explicitly carried out to the first order in ω^{-1}. Expressions for the drifts which arise in the first order are also obtained."

M. L. Goldberger (Princeton, N.J.)

4352:

Chandrasekhar, S.; Kaufman, A. N.; Watson, K. M. The stability of the pinch. Proc. Roy. Soc. London. Ser. A. **245** (1958), 435–455.

In a previous paper [#4350] the authors derived equations for small departures from a stationary state of a plasma in a strong magnetic field. These results are now employed to investigate the stability of a cylindrical plasma in an axial magnetic field and surrounded by cylindrical walls. The starting point is Boltzmann's equation without collision term and departures proportional to $\exp(ikz+im\theta+\Omega t)$ are investigated. The relevant parameters describing the unperturbed stationary state are the diameter of the plasma, the relative magnetic field strengths inside and surrounding the plasma, and the ratio of transverse and longitudinal pressures. The condition $\Omega=0$ yields those values of these parameters which separate regions of stability from regions of instability. Numerical results are given. The paper cannot be read without the above-mentioned previous one.

N. G. van Kampen (Utrecht)

4353:

Rudakov, L. I.; Sagdeev, R. Z. Oscillations of an inhomogeneous plasma in a magnetic field. Ž. Èksper. Teoret. Fiz. **37** (1959), 1337–1341 (Russian. English summary); translated as Soviet Physics. JETP **10** (1960), 952–954.

Authors' summary: "Small oscillations of a hot plasma contained by a magnetic field are treated by means of the kinetic equation in the 'drift' approximation without the collision integral. Two waves can be excited when the propagation vector is in the plane perpendicular to the direction of the unperturbed magnetic field: a slow (drift) wave with a propagation velocity of the order of the mean drift velocity of the electrons (ions) in the unperturbed state, and a magnetoacoustic wave. The first is found in an inhomogeneous plasma only. If certain relations obtain between the zeroes of the magnetic field gradients, the plasma density, and the temperature, the drift current can cause amplification of these waves. Criteria for an instability of this kind are obtained."

4354:

Kovrižnyh, L. M. Velocity distribution of electrons in a strong electric field. Ž. Èksper. Teoret. Fiz. **37** (1959), 1394–1400 (Russian. English summary); translated as Soviet Physics. JETP **10** (1960), 989–993.

Author's summary: "A method is developed for finding a nonstationary solution of the Boltzmann equation in the case of strong electric fields. An expression is derived for the electron distribution function in a completely ionized plasma located in a strong electric field. It is shown that in the first approximation the distribution is a Maxwellian one superimposed on the general translational motion of the electron gas. In the first approximation the translational velocity increases proportionally with time, whereas the temperature remains constant."

4355:

Kadomcev, B. B. Convective pinch instability. Ž. Èksper. Teoret. Fiz. **37** (1959), 1096–1101 (Russian); translated as Soviet Physics. JETP **10** (1960), 780–783.

Stability limits of axially symmetric plasmas with no longitudinal magnetic field are considered. They are obtained by applying a variational principle to two forms of the potential energy, one applicable to a hydrodynamic model and one to a kinetic model. Insufficient details are given to follow the derivations.

W. P. Allis (Cambridge, Mass.)

4356:

Green, H. S. Ionic theory of plasmas and magnetohydrodynamics. Phys. Fluids **2** (1959), 341–349.

This paper gives the dielectric and diamagnetic theory of plasmas. Free charges $e_a n_a$ are distinguished from the bound charges $\varepsilon=\sum_b e_b(n_{ab}-n_a n_b)/n_a$, where n_{ab} is the correlation density, and similarly for the currents. This introduces a polarization vector P and a distinction between E and D which other authors are criticized for not making. One reason other authors do not use this representation is that sheath charges, although they are indeed "bound", are not proportional to the applied field, and hence a laboratory plasma does not behave as a conducting dielectric. However, the author deals with plasmas which are close to thermal equilibrium, and this excludes the regions of plasmas near sheaths. Magnetohydrodynamic equations are obtained which differ, somewhat, from those of Spitzer. *W. P. Allis* (Cambridge, Mass.)

4357:

Sturrock, P. A. Excitation of plasma oscillations. Phys. Rev. (2) **117** (1960), 1426–1429.

Author's summary: "The theory of Bohm and Gross and the experiments of Looney and Brown upon the excitation of plasma oscillations by the two-stream mechanism, which appear superficially to be in disagreement, are here shown to be compatible with each other and with related experiments."

The dispersion relation appropriate to a thin beam is given. *W. P. Allis* (Cambridge, Mass.)

4358:

Cavaillès, Paul; Jancel, Raymond; Kahan, Théo. Mécanique statistique des plasmas à plusiers composantes. C. R. Acad. Sci. Paris **250** (1960), 3282–3284.

Authors' summary: "On déduit les équations macroscopiques d'un plasma à plusieurs composantes (chargées et neutres) des équations de Boltzmann. Les résultats sont appliqués aux équations phénoménologiques de plasmas binaire et ternaire."

4359:
Cavaillès, Paul; Jancel, Raymond; Kahan, Théo. Magnétohydrodynamique d'un plasma ternaire et types d'onde associés. C. R. Acad. Sci. Paris **250** (1960), 3789–3791.

Authors' summary: "On utilise les résultats de la Note précédente [see preceding review] pour établir les équations de l'hydromagnétisme d'un plasma ternaire. En linéarisant ces équations (oscillations de faible amplitude), on obtient l'équation de dispersion du milieu qui permet d'analyser les divers modes de propagation d'ondes."

4360:
Kadomcev, B. B. Equilibrium of a plasma with helical symmetry. Ž. Èksper. Teoret. Fiz. **37** (1959), 1352–1354 (Russian. English summary); translated as Soviet Physics. JETP **10** (1960), 962–963.

Author's summary: "A simple example of plasma equilibrium in a magnetic field with helical symmetry is considered."

4361:
Gordeev, G. V. Influence of boundaries on plasma oscillation. Zap. Leningrad. Gorn. Inst. **36** (1958), no. 3, 74–84. (Russian)

4362:
Breit, G.; Gluckstern, R. L. Effect of finite life of upper level on probability of Coulomb excitation. Nuclear Phys. **20** (1960), 188–201.

Authors' summary: "The effect of the finite life of the upper level on the probability of Coulomb excitation is calculated in the semiclassical approximation. The problem is first treated by considering the upper level to be coupled to the continuum by means of a matrix element as in the Weisskopf-Wigner treatment of emission and absorption in radiation theory. Equations for the excitation probability at intermediate times are worked out. The overall transition probability from the ground state to the continuum is transformed in terms of an integral over energies for a transition to a level with infinite life time. The result is then interpreted and generalized in terms of direct transitions to the continuum of stationary states without making use of the division of the nuclear Hamiltonian into parts which provided the matrix element of the first treatment. The second approximation provides a more accurate formula for the overall transition probability and relates the Coulomb excitation probability directly to the Einstein absorption probabilities per unit frequency range. A schematic illustration is given for the increase in the number of transitions from the ground state."

4363:
Ландау, Л. Д. [Landau, L. D.]; Лифшиц, Е. М. [Lifšic, E. M.]. Теория поля [Field theory]. Theoretical Physics, Vol. 2. 3rd revised edition. Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1960. 400 pp. 9 r.

This book, the previous edition of which had been already translated into English as *The classical theory of fields* [Addison-Wesley, Cambridge, Mass., 1951; MR **13**, 289], belongs to the celebrated textbook series of the authors. It deals with questions of the electromagnetic field and the gravitational field—a curious yet appropriate combination of subject matters. The presentation is extremely clear and comprehensive. Not only are the basic principles worked out in great detail, but also many important applications find their place in the text and in the well chosen problems. The elaborate study of the book should be highly recommended to all graduate students in theoretical physics, because it not only covers electromagnetic theory and relativity, but also gives an excellent introduction to classical field theory in general.
P. Roman (Boston, Mass.)

4364:
Fodor, G. The interpretation of characteristics and of fundamental equations of the electromagnetic field. Period. Polytech. Elec. Engrg. **3** (1959), 197–215.

4365:
Builder, Geoffrey. Electrodynamics. Bull. Inst. Phys. **1958**, 311–313.

It is shown that Grassman's formula

$$\mathbf{F} = (\mu_0/4\pi r^3) ii' \mathbf{ds}'(\mathbf{ds} \times \mathbf{r})$$

for the force between current elements ids, $i'ds'$ can be derived from Coulomb's law on the basis of special relativistic mechanics, whereas the Ritz and Ampere formulae are incompatible with the latter. $(v/c)^4$ and the ratio of carrier drift to gross velocities are neglected vs.1 in the proof. *H. G. Baerwald* (Albuquerque, N.M.)

4366:
Bennett, Willard M. Störmer orbits. Proc. Sympos. Appl. Math., Vol. 9, pp. 19–28. American Mathematical Society, Providence, R.I., 1959.

A description of orbits of a charged particle in a magnetic field which are of importance for cosmic ray studies. These orbits were studied with an analogue computer and several photographs indicating complicated configurations are given. *J. Moser* (Cambridge, Mass.)

4367:
Nikol'skiĭ, V. V. Slow waves in a gyrotropic medium. Radiotehn. i Élektron. **5** (1960), 39–45 (Russian); translated as Radio Engrg. and Electronics **5**, no. 1, 55–65.

Author's summary: "This article demonstrates that sufficiently slow waves in gyro-magnetic rods (free, placed in tubes, etc.) are subject to an equation given by Walker in Phys. Rev. (2) **105** (1957), 390–399, i.e., they can be considered as 'magnetostatic'. The superimposition of oppositely directed waves causes resonance oscillations, the spectrum of which lies in the limits of the spectrum of oscillations investigated by Walker."

4368:

Ulinič, F. P. The theory of horns. Radiotehn. i Èlektron. **4** (1959), 792–798 (Russian); translated as Radio Engrg. and Electronics **4**, no. 5, 82–91.

Author's summary: "The propagation of electromagnetic waves in a plane horn with a fairly small flare angle is examined. The sectoral part of the horn is taken to be infinite, and therefore no allowance is made for effects connected with the open end. The method of solution used enables the effects of the reflection and scattering of the waves to be calculated to an accuracy of any power of the minor parameter.

"It is shown that the reflection and scattering depend in a substantial way on the smoothness of the coupling. The fields and the reflection coefficient are found in the lowest order in terms of the minor parameter. The method admits a simple generalization to the three-dimensional case."

4369:

Gercenštein, M. E.; Kinber, B. E. On the electrodynamics of a resonator containing a gyrotropic medium with variable parameters. Radiotehn. i Èlektron. **5** (1960), 150–161 (Russian); translated as Radio Engrg. and Electronics **5**, no. 1, 216–232.

Authors' summary: "The quadratic relations for a resonator with a medium having time-dependent parameters are generalized. The case of a gyrotropic dispersive medium is considered. Equations are obtained for the normal wave amplitudes. The magnetostatic solutions are considered from the general electrodynamic point of view. The energy relations for the magnetostatic solutions are derived."

4370:

Haskind, M. D. Excitation of surface electromagnetic waves in plane dielectric layers. Radiotehn. i Èlektron. **5** (1960), 188–197 (Russian); translated as Radio Engrg. and Electronics **5**, no. 2, 14–28.

Author's summary: "A conducting plane covered by a dielectric film is considered and the electromagnetic field over this plane excited by given sources is investigated. Simplified boundary conditions at the surface of the dielectric layer are adopted to solve the problem and a method is developed for determining the complete wave field by means of which the formation of surface waves can be simply carried out. The general method relates to the analysis of surface wave excitation by electric and magnetic dipoles and their distributions. The results obtained may be used to improve calculation of plane surface wave antennas."

4371:

Štepa, N. I. Numerical determination of the trajectory of a charged relativistic particle in electric and magnetic fields. Ž. Tehn. Fiz. **30** (1960), 121–124 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 109–112.

Author's summary: "The trajectory of a charged relativistic particle in electric and magnetic fields is determined by applying a numerical extrapolation method directly to the system of second-order differential equations. Recursive relations for error bounds are given. An example of the numerical determination of a trajectory is also given." *W. Gautschi* (Oak Ridge, Tenn.)

4372:

Gaĭduk, V. I. On calculating induced current caused by arbitrary motion of charged particles. Radiotehn. i Èlektron. **5** (1960), 239–254 (Russian); translated as Radio Engrg. and Electronics **5**, no. 2, 90–111.

Author's summary: "Starting from the Ramo-Shockley theorem for the current induced by a single charge, expressions are obtained for the induced current with arbitrary given motion of many charged particles continuously introduced into the system. These expressions are simple, if in exit from the system the particles do not pass each other. In the presence of crossing, however, the expressions for the Fourier components of induced current may be simple for periodic or non-periodic time dependence of induced current.

"These conclusions are also applicable to charge density and convection current density (at a point) and in general, to arbitrary additive effects of any objects if the effect induced in the system by each object is known.

"As an example we consider the idealized circuits of a retarding-field oscillator and a monotron for which the electron efficiency and induced current harmonics are found. It is demonstrated that the first system has appreciably greater efficiency than the second, with complete identity of form of the fundamental characteristics and the corresponding mathematical expressions."

4373:

Beaufays, O. Sur la perte par courants de Foucault dans un noyau cylindrique en basse fréquence. Acad. Roy. Belg. Bull. Cl. Sci. (5) **45** (1959), 870–875.

Author's summary: "On établit une formule générale donnant la perte par courants de Foucault dans un noyau cylindrique de section arbitraire, en basse fréquence. Cette perte est indépendante de la perméabilité du matériau. Elle n'est donc pas influencée par une variation de la perméabilité en fonction du champ magnétique."

4374:

Beaufays, O. Sur la perte par courants de Foucault dans un noyau cylindrique doux, dans l'état quasi-stationnaire. Acad. Roy. Belg. Bull. Cl. Sci. (5) **45** (1959), 1078–1083.

Author's summary: "On établit une formule générale donnant la perte par courants de Foucault, dans un noyau cylindrique de section arbitraire, magnétique doux, et soumis à une induction sinusoïdale B, l'état étant quasi-stationnaire."

4375:

Skobelkin, V. I. On magnetic flux functions for a three-dimensional field. Dokl. Akad. Nauk SSSR **128** (1959), 280–283 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 1007–1010.

A short general article dealing with a variational principle for a static, three-dimensional magnetic field expressed in the form $\mathbf{B}=\nabla\psi\times\nabla\theta$, where ψ and θ are a pair of magnetic flux functions. A stationary integral

involving these flux functions is constructed from the Lagrangian of the field, and appropriate partial differential equations and boundary conditions are derived. Questions of uniqueness and of the convergence of minimizing Ritz sequences are discussed. No examples are given. *R. D. Kodis* (Providence, R.I.)

4376:

Hammond, P. Electric and magnetic images. Proc. Inst. Elec. Engrs. C **107** (1960), 306–313.

Author's summary: "The method of images as applied to electrostatic, magnetostatic and electromagnetic fields is investigated. By considering the uniqueness of the field it is shown within what limits the method can safely be used, and rules are given for its use. The application of the method is illustrated by a discussion of the electric field near a cylindrical cathode and the magnetic fields near the end-windings of electrical machines."

4377:

Peligry, Claude. Calcul numérique de la répartition du potentiel d'une grille disposée entre deux plaques parallèles. C. R. Acad. Sci. Paris **250** (1960), 4108–4110.

4378:

Durandeau, P. Étude sur les lentilles électroniques magnétiques. Ann. Fac. Sci. Univ. Toulouse (4) **21** (1957), 1–87 (1959).

4379:

Brown, William Fuller, Jr. Single-domain particles: new uses of old theorems. Amer. J. Phys. **28** (1960), 542–551.

Author's summary: "This article describes some applications of potential theory and classical dynamics to one field of current research, the theory of magnetization processes in 'hard' magnetic materials. Many such materials owe their magnetic properties to the fact that they are composed of fine ferromagnetic particles separated by less magnetic regions. In very fine particles, the exchange forces responsible for the spontaneous magnetization keep it uniform in direction within any one particle; the magnetization curve is then determined by rotations of the particle magnetizations. The rotations are controlled by the interplay of external magnetic energy and of internal energy, also largely magnetic. A first step in the theory is therefore the derivation of a general formula for the internal magnetic energy. Simplifications can then be made because of the uniformity of magnetization of each particle; in particular, it can be shown that a particle of arbitrary shape is equivalent to an ellipsoid. Calculation of static hysteresis loops requires consideration of stability conditions; analysis of magnetization reversals and of the behavior in microwave fields involves the dynamics of gyroscopic systems."

4380:

Waldron, R. A. Perturbation theory of resonant cavities. Proc. Inst. Elec. Engrs. C **107** (1960), 272–274.

Author's summary: "A detailed derivation is given of the perturbation formula for the frequency shift on introducing a sample of ferrite or dielectric material into a resonant cavity. The purpose of this is to make clear what assumptions are involved in the derivation; it is necessary to appreciate what these assumptions are in order to design accurate experiments."

4381:

Bolotovskiĭ, B. M.; Ruhadze, A. A. Field of a charged particle in a moving medium. Ž. Èksper. Teoret. Fiz. **37** (1959), 1346–1351 (Russian. English summary); translated as Soviet Physics. JETP **10** (1960), 958–961.

Authors' summary: "The field produced by the motion of a charge through a moving medium is considered. The energy losses of the charge due to Čerenkov radiation and excitation of plasma waves are determined."

4382:

Hevrunin, I. S. Calculation of the radiation efficiency of an aerial having a radiation pattern of ellipsoidal form. Radiotehn. i Èlektron. **4** (1959), 381–383 (Russian); translated as Radio Engrg. and Electronics **4**, no. 3, 41–44.

Author's summary: "Formulae are derived for the calculation of the radiation efficiency of aerials, the field-strength radiation patterns of which can be represented by triaxial ellipsoids having one of the axes orientated in the direction of maximum radiation."

4383:

Vahman, D. E. Application of the stationary phase principle to the calculation of radio pulse spectra. Radiotehn. i Èlektron. **4** (1959), 1124–1133 (Russian); translated as Radio Engrg. and Electronics **4**, no. 7, 78–92.

Author's summary: "An approximate calculation of the spectrum of bell-shaped pulses with carrier frequency modulation is given, based on the stationary phase principle. A physical interpretation of the method is given and its connexion with the 'method of instantaneous frequency' is shown. A modification of the stationary phase method is proposed. The case is considered when the laws of the frequency and pulse amplitude variations are identical."

4384:

Budden, Kenneth G. The "waveguide mode" theory of the propagation of very-low-frequency radio waves. Proc. IRE **45** (1957), 772–774.

4385:

Olving, Sven. Electromagnetic and space charge waves in a sheath helix. Chalmers Tekn. Högsk. Handl. No. 225 (1960), 88 pp.

A field analysis is presented of a circularly cylindrical electron beam confined by an infinite dc magnetic field and surrounded by a sheath helix. The case of large beam radius is covered and the limiting forms of the results for a thin beam are checked with previous work. All higher order space charge waves are taken into account. Only one pair of complex roots is found in the interaction region when the helix wave is near synchronism with the

space charge waves. The higher order space charge modes are perturbed by the helix without giving rise to complex roots.

In the range of beam velocities for which complex propagation constants are found, the graphs of these propagation constants versus beam velocity check with the results of the circuit theory of the TWT as developed by Pierce. Just outside this range, at the low beam velocity end, the presence of the higher space charge waves leads to graphs quite different from those obtained from an analysis disregarding these waves (one-dimensional analysis or circuit analysis).

The range of velocities over which amplification occurs is found to be wider than previously assumed. At high current densities thick beams have a maximum amplification that is independent of the current density. Hollow beams of nonzero thickness are also treated.

H. A. Haus (Cambridge, Mass.)

4386:

Vaĭnštein, L. A. Radar detection of a "fluctuating target" in the presence of correlated noise. I. A coherent packet of signals. Radiotehn. i Èlektron. **4** (1959), 735–744 (Russian); translated as Radio Engrg. and Electronics **4**, no. 5, 1–15.

Author's summary: "A theoretical examination is made of the optimum radar receiver for the detection of a packet of coherent signals reflected off a 'fluctuating target', i.e., signals subjected to fluctuation as a result of small changes in the orientation of a complex reflecting target. Detection occurs in the presence of normal interference having intraperiodic (for the duration of each signal) and interperiodic (for the duration of a whole packet) correlations. The coefficient of likelihood is calculated, and the optimum intraperiodic and interperiodic processing of the mixture of useful signals and noise appearing at the receiver input, is examined. The probabilities of spurious and correct detection are calculated for a receiver realizing interperiod subtraction."

4387:

Vaĭnštein, L. A. Radar detection of a "flickering target" in the presence of correlated noise. II. Incoherent packet of signals. Radiotehn. i Èlektron. **4** (1959), 1071–1078 (Russian); translated as Radio Engrg. and Electronics **4**, no. 7, 1–12.

Author's summary: "As a supplement to the first part of the work we have investigated the optimum radar receiver set up for the purpose of detecting a 'flickering target' by an incoherent packet of signals in the presence of internal noises or other normal interference which has some arbitrary interperiod correlation but which is not correlated from one signal repetition period to another. Exact and approximate expressions for the probability of false alarm and genuine detection with square law accumulation have been derived. The characteristics of detection of a 'rapidly' and a 'slowly' flickering target have been given."

4388:

Zaborov, V. P. The method of isometric transformation of radio lenses. Radiotehn. i Èlektron. **4** (1959), 576–583 (Russian); translated as Radio Engrg. and Electronics **4**, no. 4, 24–35.

Author's summary: "A method of constructing and calculating radio lenses with variable index of refraction is presented in this paper.

"It is shown how, utilizing the results of calculation of one lens, it is possible by the use of an isometric transformation, i.e., a transformation, an invariant of which is the element of optical path in the medium with variable index of refraction, to obtain new lenses which may differ from the original in both the form of the body and the form of the phase front.

"The isometric transformation represents a conformal representation of the plane of the lens profile. The index of refraction is transformed according to the formula $\bar{N}_i = N_i/|w^{*\prime}|$, where $|w^{*\prime}| = |dw^*/dz^*|$ is the modulus of the derivative of the complex analytic function $w^*(z^*)$, while $i = 1, 2, 3, \cdots$ is the index of the section of the profile plane in the limits of which N_i is a continuous function of the co-ordinates."

4389:

Markovkin, A. M. Some properties of media with periodically time-varying parameters ε and μ. Radiotehn. i Èlektron. **4** (1959), 785–791 (Russian); translated as Radio Engrg. and Electronics **4**, no. 5, 72–81.

Author's summary: "The concept of parametric phenomena is extended to media with variable parameters. The emission (absorption) conditions are formulated for electromagnetic waves propagated or localized, in the absence of external electric and magnetic forces, in media with time-varying parameters. Examples of the design of some systems based on the application of the energy relationships are given."

4390:

Funk, J. P. Behavior of freely exposed absorbers in radiation fields. J. Opt. Soc. Amer. **50** (1960), 986–991.

Author's summary: "General formulas for the absorption, emission, and radiative equilibrium temperature of arbitrary absorbers are derived and in particular those of slabs and spheres in atmospheric long-wave radiation fields. The behavior of all nongaseous absorbers in these radiation fields is shown to be rather similar and similar again to that of a black sphere. The optical behavior of moist air, on the other hand, is shown to be completely different because of its great transparency in the 'window' wavelengths. Radiation instruments of the 'blackball' type can therefore neither be used for the measurement of the radiative equilibrium temperature of air nor for the divergence of the atmospheric long-wave flux."

4391:

Pistol'kors, A. A.; Kaplun, V. A.; Knyazeva, L. V. On the diffraction of electromagnetic waves at dielectric or semi-conducting sheets. Radiotehn. i Èlektron. **4** (1959), 911–919 (Russian); translated as Radio Engrg. and Electronics **4**, no. 6, 1–13.

Authors' summary: "Diffraction at a dielectric or semiconducting sheet is investigated by Kirchhoff's method and expressions are derived valid for points sufficiently far from the edges and the surface of the sheet. Certain particular cases of diffraction are considered. The results of experimental verification are presented."

4392:

Siu, Yen-Sheng. The diffraction of electromagnetic waves on a gyrotropic sphere inside a rectangular waveguide. Radiotehn. i Èlektron. **5** (1960), 15–26 (Russian); translated as Radio Engrg. and Electronics **5**, no. 1, 18–34.

Author's summary: "The problem of diffraction of electromagnetic waves on a gyrotropic sphere situated inside a rectangular waveguide, in which waves of basic type H_{10} are propagated, is studied. Expressions are obtained for the resolution of normal type H_{10} waves into spherical waves for the two cases when the direction of the constant magnetic field is parallel with either the wider or the narrower side-wall of the waveguide. The amplitudes of the waves reflected from the sphere are calculated and the problem of excitation of the waveguide by the reflected waves is solved."

4393:

Peresada, V. P. Diffraction in a non-uniform field. Radiotehn. i Èlektron. **4** (1959), 384–387 (Russian); translated as Radio Engrg. and Electronics **4**, no. 3, 45–50.

Author's summary: "A method for obtaining an approximate solution of the problem of non-uniform electromagnetic field diffraction by perfectly conducting bodies is given.

"The particular problem of field diffraction by a rectangular plate situated above the sphere of the earth is studied.

"Formulae for the calculation of the electric field potential of the secondary radiation from the plate are given in terms of the attenuation functions $v(x, y_1, y_2, q)$, which were extensively studied and tabulated by Academician V. A. Fok."

4394:

Pistol'kors, A. A.; Siu, Yen-Sheng. The oscillations of a small gyrotropic sphere in the field of a plane wave. Radiotehn. i Èlektron. **5** (1960), 3–14 (Russian); translated as Radio Engrg. and Electronics **5**, no. 1, 1–17.

Authors' summary: "The excitation of incident plane-waves of electro-magnetic oscillations in a gyrotropic sphere of small radius and the diffraction phenomena entailing it are investigated. In calculating the magnetic field the solutions of Walker are used as a basis. The dependence of the amplitude and the resonance curve of the oscillations upon various factors is studied. The value obtained for the magnetic induction is employed in determining the electric field. As a result of the gyrotropic character of the sphere a transformation of the types of oscillations appears to be possible: the Debye potential of the diffracted waves might differ in its type from the potential of the energizing waves."

4395:

Bahareva, M. F. The correlation between waves of different frequencies travelling through a layer of statistically inhomogeneous medium. Radiotehn. i Èlektron. **4** (1959), 88–96 (Russian); translated as Radio Engrg. and Electronics **4**, no. 1, 141–155.

Author's summary: "The correlation coefficients at one and the same point are obtained for the fluctuations of the levels (the logarithms of the amplitudes) and of the phases of two waves of different frequencies, travelling through partially overlapping layers of a medium having large-scale random inhomogeneities of refractive index. The results are compared with experimental data on frequency-diversity sounding of the ionosphere. The estimate thus obtained for the dimensions of the inhomogeneities in the F and E layers agrees with values found from space-diversity correlation data."

4396:

Rodak, M. I.; Frantsesson, A. V. On the use of turbulence theory to investigate the scattering of radio waves from wandering inhomogeneities. Radiotehn. i Èlektron. **4** (1959), 398–403 (Russian); translated as Radio Engrg. and Electronics **4**, no. 3, 66–74.

Authors' summary: "Scattering of radio waves from particles, the motion of which can be described in terms of Kolmogorov and Obukhov's turbulence theory, is investigated.

"The conditions are clarified, which make it permissible to evaluate the correlation function (the spectrum) of the scattered field (of the components of its amplitude) and the correlation function (the spectrum) of its intensity, while assuming the velocity of the scattering centres to be constant.

"The field spectrum (the spectrum of the components of its amplitude) and the intensity spectrum are compared."

4397:

Howells, I. D. The multiple scattering of waves by weak random irregularities in the medium. Philos. Trans. Roy. Soc. London. Ser. A **252** (1960), 431–462.

The first part of this paper reviews the single-scattering theory for the scattering of a wave by a medium with weak random inhomogeneities. The plane-wave scattering coefficient is derived in terms of the correlation function of the inhomogeneities, and the limits of validity of the theory found, for the cases of: sound waves and varying sound speed and density; sound waves in a turbulent medium; electromagnetic waves and varying permittivity. The conditions are derived for which the multiple scattering cases can be described by an equation of transfer. In Part II, the author first considers the solution to the equation of transfer for a time-dependent spatially-homogeneous field. The major portion of this section concerns the steady-state spatially-varying radiation field. An approximate solution is derived for small scattering angle and small total angular deviation. For the case of small scattering angle but large total angular deviation, an approximate equation is derived; methods of solution and some numerical results are given.

D. Mintzer (New Haven, Conn.)

4398:

Fejer, J. A. Scattering of radio waves by an ionized gas in thermal equilibrium. Canad. J. Phys. **38** (1960), 1114–1133.

The power scattered by density fluctuations in an ionized gas is calculated, assuming the presence of discrete simply charged positive ions which neutralize the

electrons. The Fourier analysis of the spontaneous density fluctuations is based on the Maxwell-Boltzmann energy distribution. It is found that the scattering cross section at large wavelengths is half that for small wavelengths (and non-zero angle between the incident and scattered waves). The frequency spectrum of the scattered power is determined for both the low and the high collision frequency approximations. It is first assumed that the scatterers execute their random thermal motion uninfluenced by fluctuating electrostatic fields and then the results are corrected for the effect of the electrostatic field on the motion of the positive ions. The expressions obtained make it possible to interpret the results of observations of this type of scattering from the ionosphere in terms of electron density and temperature.

J. A. Morrison (Murray Hill, N.J.)

4399:

Grinberg, G. A. Diffraction of electromagnetic waves by strip of finite width. Dokl. Akad. Nauk SSSR **129** (1959), 295–298 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 1222–1226.

Continuing his earlier work on the method of shadow currents [Z. Tehn. Fiz. **28** (1958), 542–554, 555–568; MR **21** #568, #569] the author shows that the problem of diffraction by a slit reduces to a Fredholm integral equation of the second kind from which asymptotic solutions may be found.

A. E. Heins (Ann Arbor, Mich.)

4400:

Yugova, G. A. On the problem of diffraction of waves on systems of particles. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 2, 121–134. (Russian)

4401:

Didenko, A. N. Propagation of electromagnetic waves in curved loaded waveguides. Radiotehn. i Èlektron. **4** (1959), 172–180 (Russian); translated as Radio Engrg. and Electronics **4**, no. 2, 24–34.

Author's summary: "The propagation of electromagnetic waves in curved waveguides loaded by diaphragms or partially filled with dielectric is considered. The possibility of the propagation of *LH*-waves with definite phase velocities is evaluated and the nonuniformity of the tangential electric field components is estimated."

4402:

Millea, Aurel. Approximate computation of the critical frequencies in waveguides with polygonal cross-section. Bul. Inst. Politehn. Bucureşti **20** (1958), no. 3, 161–172. (Romanian. Russian, English, French and German summaries)

Author's summary: "One presents a method for the approximate computation of the critical frequencies, founded on a variational principle. The method makes use of a function $u(x, y)$ which approximates the transversal variation of the electromagnetic field. This function will satisfy the condition on the boundary but will not satisfy exactly the wave equation. Indications are given concerning the function $u(x, y)$ in polygonal waveguides. The advantage of the method lies in the fact that relatively simple formulae are obtained and no numerical computing is necessary. The case of a triangular waveguide is worked out; the results are in good agreement with the exact values previously known for certain particular cases."

4403:

Kogan, S. H. Attenuation of electromagnetic waves propagating along a helical wire line. Radiotehn. i Èlektron. **4** (1959), 181–186 (Russian); translated as Radio Engrg. and Electronics **4**, no. 2, 35–42.

Author's summary: "The characteristic equation determining the propagation constant in a helical line composed of a circular conductor with finite conductance is derived and its dependence on the helix geometry and wavelength is examined. From the approximate solution of the equation, formulae are obtained for calculating the phase velocity and attenuation. The behaviour of the attenuation over a wide frequency band is illustrated by an example of a practical helical line."

4404:

Gercenštein, M. E.; Funtova, N. F. Polarization isolators in a waveguide. Radiotehn. i Èlektron. **4** (1959), 805–813 (Russian); translated as Radio Engrg. and Electronics **4**, no. 5, 101–113.

Authors' summary: "With the propagation of electromagnetic energy in a waveguide there occurs a reciprocal transformation of the waves of the different polarization, dependent on the inexactitude of the manufacture of the walls. The reciprocity of the polarization isolators is proved. Formulae are given, connecting the polarization isolators in a regular waveguide with the deviations of the form from the ideal, and graphs are given for polarization isolators. It is shown that, in order to increase polarization decoupling, it is necessary to use more rigid tolerances, to use waveguide sections at which these tolerances are guaranteed (for example, round), and to increase the transverse dimensions of the waveguide."

4405:

Tartakovskiĭ, L. B. Side radiation from ideal paraboloid with circular aperture. Radiotehn. i Èlektron. **4** (1959), 920–929 (Russian); translated as Radio Engrg. and Electronics **4**, no. 6, 14–28.

Author's summary: "The relationship is considered between the current and aperture methods of calculating parabolic antennas and the limits of applicability of the latter are established. The magnitudes of side and back radiation of an axially symmetrical antenna are calculated, the fine structure of side radiation is investigated including the character of the directional phase diagrams."

4406:

Gercenštein, M. E.; Vasil'ev, V. B. The diffusion equation for statistically inhomogeneous waveguide. Radiotehn. i Èlectron. **4** (1959), 611–617 (Russian); translated as Radio Engrg. and Electronics **4**, no. 4, 74–84.

Authors' summary: "It is shown that with regard to multiple reflections and attenuation of incident waves the probability density W for the reflection coefficient in a

waveguide (long line) with a single wave mode satisfies the diffusion equation $D\Delta W = \delta W/\delta z$, where the Laplace operator Δ is defined by a Lobachevskii-plane metric. An exact solution is found for the probability density in the case where a matched load is located at the end of the waveguide. The additional attenuation in the line due to losses of energy through reflection from random inhomogeneities is calculated. The energy incident on the terminal load is estimated after multiple reflections, constituting the cause of cross-modulation distortion of the transmitted signal."

4407:
Horiuchi, Kazuo. Electromagnetic waves in nonideal waveguides. Mem. School Sci. Engrg. Waseda Univ. Tokyo. No. 23 (1959), 80–99.

4408:
Millea, Aurel. Triangular waveguides. Bul. Inst. Politehn. Bucureşti **20** (1958), no. 4, 171–185. (Romanian. Russian, English, French and German summaries)

Author's summary: "The exact formulae for the components of the electromagnetic field and the critical frequencies are given for waveguides having cross sections of isosceles rectangular triangle, rectangular triangle with one angle of 30° and equilateral triangle shapes."

4409:
Coste, Jean; Dagens, Luc. Équilibre statistique d'une charge d'espace électronique à symétrie cylindrique dans un tube du type magnétron. C. R. Acad. Sci. Paris **250** (1960), 3009–3011.

Authors' summary: "Une théorie statistique du magnétron a été donnée par Gabor [Proc. Roy. Soc. London. Ser. A **183** (1945), 436–453; MR **7**, 104] en 1945. Le calcul a été fait en postulant un désordre partiel du faisceau électronique (pas de désordre azimutal). Nous considérerons, dans notre étude, que tous les degrés de liberté sont thermalisés. L'influence du champ magnétique sur la distribution d'équilibre sera obtenue en s'imposant, à l'équilibre, non seulement la valeur de l'énergie totale, mais aussi celle du moment par rapport à l'axe du magnétron de la quantité de mouvement totale."

4410:
Harrison, Charles W., Jr.; King, Ronold W. P. Response of a loaded electric dipole in an imperfectly conducting cylinder of finite length. J. Res. Nat. Bur. Standards Sect. D **64D** (1960), 289–293.

4411:
Hersch, W. The surface-wave aerial. Proc. Inst. Elec. Engrs. C **107** (1960), 202–212.

From the author's summary: "The theory of the surface-wave aerial is developed in detail, a necessary preliminary step being a full theoretical analysis of the properties of the first-order cylindrical surface wave. It is shown that a dielectric-coated cylinder which is approximately a wavelength in circumference can act as a waveguide for higher-order surface waves, of which the first order is an example.

"The 'characteristic equation' is determined for the general case from which the cut-off frequency, propagation coefficient and conditions under which propagation can take place are derived in turn."

4412:
Schmitt, Hans J. Transients in cylindrical antennae. Proc. Inst. Elec. Engrs. C **107** (1960), 292–298.

Author's summary: "The transient response of the radiation field of a driven cylindrical antenna is investigated for the particular case of a step-function excitation. The theoretical analysis makes use of Fourier's theorem to express the response as an integral over the response to all individual frequency components. The response as a function of time shows damped oscillations with a frequency determined by the first resonance frequency of the antenna. The response of the same antenna used as a receiver in a transient plane wave field is shown to be related to the radiation response by a simple integration process. By proper loading of the dipole, transient times of the order of the time needed for a wave to travel along the dipole axis can be obtained. An experimental investigation is described in which the reception of a transient field due to a shock-excited distant transmitter is observed."

4413:
Grebenščikov, V. N. An algebraic method of analysis of multipolar contact-valve scheme. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 2, 69–78. (Russian)

4414:
Cederbaum, I. Voltage and current transformation matrices. Proc. Inst. Elec. Engrs. C **107** (1960), 145–149.

Author's summary: "Transformation matrices relating two adequate systems of simple network co-ordinates such as node-pair voltages or link currents belong to the class of unimodular or E-matrices. The paper makes a distinction between the matrices corresponding to such voltage and current transformations, and different sets of necessary conditions are derived for each type.

"Since the loop- and cut-set-to-branch incidence matrices are closely related to transformation matrices, the discussion proposes new sets of necessary conditions for incidence matrices corresponding to systems of node-pair voltages or link currents. Examples are given of matrices which whilst representing a voltage transformation cannot represent a current transformation and vice versa. Another example shows an E-matrix which can represent neither a voltage nor a current transformation."

4415:
Lewin, L. Radiation from discontinuities in strip-line. Proc. Inst. Elec. Engrs. C **107** (1960), 163–170.

Author's summary: "A method of calculation, which should be adequate for strip-lines of small spacing, is

proposed for the evaluation of the radiation of line-above-ground configurations. It is shown that an open-circuit is appreciably worse, from the point of view of radiation loss, than a short-circuit with a right-angle corner at an intermediate level. The radiation from a matched post is slightly worse than from the short-circuit, but it should be capable of considerable improvement by the use of a terminating frame antenna. Formulae are also given for the effects of reactive posts, with applications to a simple resonator."

4416:

Huey, R. M.; Pawloff, O.; Glucharoff, T. Extension of the dual-input describing-function technique to systems containing reactive non-linearity. Proc. Inst. Elec. Engrs. C **107** (1960), 334–341.

From the authors' summary: "Kochenburger's describing function and also the dual-input describing function due to West, Douce and Livesley are well known. These techniques enable the graphical solution of non-linear differential equations in which the non-linear coefficient is associated with the first-order differential term to be obtained. The paper describes an extension of these methods, allowing the non-linear coefficient to be associated with any term in the differential equation."

4417:

Brosan, G. S. A new form of tensor equations of electrical machines. Proc. Inst. Elec. Engrs. C **107** (1960), 299–305.

Author's summary: "Previous tensor methods applied to electrical engineering systems have been based on Lagrange's equations, which are not always suitable for non-holonomic systems. As a consequence, the defining equations lose much of their simplicity and the component terms in the equations may no longer be tensors.

"The paper presents a new set of equations based on the principle of least curvature. These equations are suitable for both holonomic and non-holonomic systems, and their application is shown by examples."

4418:

Dymčišin, V. N. Distribution of pulse durations at coincidence-circuit output. Radiotehn. i Èlektron. **4** (1959), 960–965 (Russian); translated as Radio Engrg. and Electronics **4**, no. 6, 70–77.

Author's summary: "The probability density distribution function and the integral function of the distribution of pulse durations at the output of a coincidence circuit are calculated for inputs of random and periodic pulse signals."

4419:

Mar'in, N. P. Butt joint of different waveguide types. Radiotehn. i Èlektron. **4** (1959), 3–11 (Russian); translated as Radio Engrg. and Electronics **4**, no. 1, 1–14.

Author's summary: "The problem concerning the incidence of electromagnetic waves on the butt joint of two plane waveguides, the walls of which are along lines of different co-ordinate systems, is solved in this article. The problem is reduced to the solution of an infinite system of equations in which the unknown quantities are the amplitudes of the transmitted and reflected waves. In the case when there is no transverse resonance at the butt joint, the solution is found by the method of successive approximations."

4420:

Samoilenko, Yu. I. Discrimination of an oscillator synchronized to a harmonic signal. Radiotehn. i Èlektron. **4** (1959), 39–42 (Russian); translated as Radio Engrg. and Electronics **4**, no. 1, 60–66.

Author's summary: "The behaviour of an oscillator synchronized to an external harmonic signal under the effect of interfering quasi-harmonic e.m.f. is analysed. Those cases are considered when a harmonic or a fluctuational interference acts on the system. An analogy is established between the synchronized oscillator and a single resonant circuit."

4421:

Gavelya, N. P. The distribution of current in an infinite cylindrical conductor in the case of lumped excitation. Radiotehn. i Èlektron. **4** (1959), 404–416 (Russian); translated as Radio Engrg. and Electronics **4**, no. 3, 75–92.

Author's summary: "The distribution of current in an infinite cylindrical conductor in the case of lumped excitation, symmetrical with respect to the axis of the cylinder, is considered.

"The conductivity of the substance of which the conductor is made is assumed to be infinite, while the parameters of the surrounding medium can be chosen arbitrarily. The distribution function of current is represented by a series, convergent for all values of the distance from the source."

4422:

Amyantov, I. N. Non-inertial envelope transformation of quasi-harmonic fluctuations. Radiotehn. i Èlektron. **4** (1959), 449–456 (Russian); translated as Radio Engrg. and Electronics **4**, no. 3, 140–149.

Author's summary: "The two-dimensional moment $[G_1(A_1)G_2(A_2)]$ is calculated for non-inertial transformations of G_1 and G_2 of the envelope of quasi-harmonic fluctuations $A_1=A(t)$, $A_2=A(t+\tau)$. Cases of step-wise linear and exponential transformations of G_1 and G_2 are considered. In two particular examples the results are illustrated by graphs."

4423:

Pokrovskiĭ, V. L.; Ulinič, F. R.; Savvinyh, S. K. The theory of tapered waveguides. Radiotehn. i Èlektron. **4** (1959), 161–171 (Russian); translated as Radio Engrg. and Electronics **4**, no. 2, 10–23.

Authors' summary: "The propagation of electromagnetic waves in plane waveguides with cross-sections varying slowly but by a finite quantity is considered. A method is adopted combining perturbation theory with the method of Wentzel-Kramer-Brillouin (WKB). The method gives the possibility of calculating in principle the effects of reflection and scattering of waves with an

accuracy to an arbitrary power of small parameter. It is shown that reflection and scattering depend essentially on the smoothness of coupling. Reflection and scattering coefficients are found in the lowest power of the small parameter. This power depends on the coupling smoothness."

4424:

Dmitriev, Yu. Reversal points in waveguides of variable section. Radiotehn. i Elektron. **4** (1959), 966–971 (Russian); translated as Radio Engrg. and Electronics **4**, no. 6, 78–85.

Author's summary: "The results of work by Pokrovskiĭ, Ulinič and Savvinyh [see preceding review] are generalized to the case of waves with reversal points, i.e., being fully reflected from some cross-section of the waveguide. It is shown that with the use of a simple procedure this generalization may be carried out for waveguides with arbitrary section."

4425:

Savvinyh, S. K. On the theory of waveguides of variable circular cross-section. Radiotehn. i Elektron. **4** (1959), 972–979 (Russian); translated as Radio Engrg. and Electronics **4**, no. 6, 86–97.

Author's summary: "The methods developed in work by Pokrovskiĭ, Ulinič and Savvinyh [see review above] in application to plane waveguides will be extended in the present article to the case of waveguides with slowly varying circular section. Local reflections and scattered waves produced by defects in the boundaries of waveguides (disturbances of smoothness, discontinuities in curvature, etc.) are considered. Expressions are found for scattered and reflected waves in the lowest approximation of a small parameter."

4426:

Stel'mah, M. F.; Ol'derogge, E. B. Propagation of electromagnetic waves in diaphragm-type delay structures with annular slots. Radiotehn. i Elektron. **4** (1959), 980–987 (Russian); translated as Radio Engrg. and Electronics **4**, no. 6, 98–109.

Authors' summary: "The article considers the propagation of electromagnetic waves in a waveguide and coaxial line, loaded by diaphragms with annular slots. The dispersion equations are found and expressions obtained for the coupling factors of the field and beam in such lines."

4427:

Sedyakin, N. M. The response of an oscillating system with linearly-varying resonant frequency. Radiotehn. i Elektron. **4** (1959), 457–462 (Russian); translated as Radio Engrg. and Electronics **4**, no. 3, 150–158.

Author's summary: "The object of the present article is to establish quantitative relationships between the rate of variation of the resonant frequency, the error in the measurement of frequency and the bandwidth of an oscillating system having a Gaussian characteristic.

"It is shown that for a given rate of variation of the resonant frequency there exists an optimum bandwidth of the system ensuring the carrier frequency of a signal to be measured with maximum accuracy."

4428:

Marinescu, M. La "résonance dynamique" dans les circuits à inductance variable, sans condensateur et son application à la construction rationnelle des moteurs synchrones sans excitaton. Com. Acad. R. P. Romîne **10** (1960), 23–31. (Romanian. Russian and French summaries)

Author's summary: "L'auteur montre qu'en vertu d'une méthode d'analyse des systèmes électro-mécaniques à inductance variable—présentée dans un travail antérieur—le processus de la 'résonance dynamique' peut avoir lieu même en l'absence d'un condensateur dans le circuit. Dans ce cas, l'inductance 'statique' variable de la bobine est compensée par l'effet de capacité de la masse en accélération dans le circuit électrique. De cette manière on obtient, lors de la transformation en travail de l'énergie électrique, un facteur de puissance unitaire, aux bornes mêmes de la bobine."

4429:

Kontorovič, M. I. On the composition of oscillator equations. Radiotehn. i Elektron. **5** (1960), 310–322 (Russian); translated as Radio Engrg. and Electronics **5**, no. 2, 187–204.

Author's summary: "An equation with a slight nonlinearity is considered, describing the behaviour of a vacuum valve oscillator with arbitrary (finite) number of degrees of freedom. Finite difference equations are derived which are employed in composing the 'abbreviated equations', as well as to find the periodic solutions. Certain particular cases are considered and the method of constructing higher approximations indicated."

CLASSICAL THERMODYNAMICS, HEAT TRANSFER

See also 4282, 4299.

4430:

Martynov, G. A. Solution of the inverse Stefan problem in the case of spherical symmetry. Ž. Tehn. Fiz. **30** (1960), 239–241 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 215–218.

The inverse Stefan problem [the author, same Ž. **25** (1955), 1754–1767; MR **19**, 710] for spherical symmetrical systems is as follows: $\partial\theta_k/\partial t=(a_k/r^2)\partial(r^2\partial\theta_k/\partial r)/\partial r$, $R_0=R(0)\leqq r\leqq R(t)$ for $k=1$ and $R(t)\leqq r\leqq\infty$ for $k=2$; $\theta_2(r,0)=f(r)>0$; $\theta_k(R,t)=0$ $(k=1,2)$ for $r=R(t)$; $\lambda_1\partial\theta_1(R,t)/\partial r-\lambda_2\partial\theta_2(R,t)/\partial r=qdR/dt$. Here $r=R(t)$ is the law of motion of the boundary between the phases. The transformations $\Theta_k(r,t)=r\theta_k(r,t)$ and $x=r-R_0$, $h(t)=R(t)-R_0$ reduce the problem to the form considered in the cited paper, to which form the method of continuation is applicable. The laws $h=vt^{1/2}$ and $h=vt$ are treated.

R. N. Goss (San Diego, Calif.)

4431:

Murakawa Katsuhisa. Theoretical solutions of heat transfer in the hydrodynamic entrance length of double pipes. Bull. JSME **3** (1960), 340–345.

4432:
Slezkin, N. A. On the application of a method of Oseen to the plane problem of the discharge of a heated gas. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 2, 39–42. (Russian)

4433:
Popov, N. N. A new method of calculation for the mixing chamber of an ejector with many jets. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 3, 29–34. (Russian)

4434:
Plesset, M. S.; Zwick, S. A. The growth of vapor bubbles in superheated liquids. J. Appl. Phys. **25** (1954), 493–500.

Authors' summary: "The growth of a vapor bubble in a superheated liquid is controlled by three factors: the inertia of the liquid, the surface tension, and the vapor pressure. As the bubble grows, evaporation takes place at the bubble boundary, and the temperature and vapor pressure in the bubble are thereby decreased. The heat inflow requirement of evaporation, however, depends on the rate of bubble growth, so that the dynamic problem is linked with a heat diffusion problem. Since the heat diffusion problem has been solved, a quantitative formulation of the dynamic problem can be given. A solution for the radius of the vapor bubble as a function of time is obtained which is valid for sufficiently large radius. This asymptotic solution covers the range of physical interest since the radius at which it becomes valid is near the lower limit of experimental observation. It shows the strong effect of heat diffusion on the rate of bubble growth. Comparison of the predicted radius-time behavior is made with experimental observations in superheated water, and very good agreement is found."

4435:
Zwick, S. A. Growth of vapor bubbles in a rapidly heated liquid. Phys. Fluids **3** (1960), 685–692.

Author's summary: "The earlier theory by Plesset and the author [see preceding review] of the growth of vapor bubbles in superheated liquids is extended to the situation in which the rate of temperature rise of the liquid is large. Numerical solutions are presented for the early stages of bubble growth for various rates of liquid temperature rise. The asymptotic behavior of a bubble is found explicitly for a temperature rise of the liquid which is linear in time. In this case the bubble radius grows initially as $t^{1/2}$, as in asymptotic solutions found previously for small rates of temperature rise, but then deviates toward a late $t^{3/2}$ variation.

4436:
Friedlander, S. K. On the particle size spectrum of a condensing vapor. Phys. Fluids **3** (1960), 693–696.

Author's summary: "The classical theories of condensation and coagulation are considered as limiting cases of a general theory of new phase formation. By making several assumptions concerning the nature of the vapor and of the cooling process, it is shown that the equation of condensation can be written in a simplified dimensionless form. The embryo size spectrum function at the end of the condensation process is a function of a number of dimensionless groups; hence, condensation can be modeled in the sense that particle size can be controlled by varying certain scale factors. An application to condensation by mixing of a hot vapor with cool air is proposed."

4437:
Plyuhin, B. I. The laws of thermal emission of a flame. Dokl. Akad. Nauk SSSR **131** (1960), 68–71 (Russian); translated as Soviet Physics. Dokl. **5**, 279–282.

4438:
Bartholomay, Anthony F. Stochastic models for chemical reactions. I. Theory of the unimolecular reaction process. Bull. Math. Biophys. **20** (1958), 175–190.

4439:
Bartholomay, Anthony F. Stochastic models for chemical reactions. II. The unimolecular rate constant. Bull. Math. Biophys. **21** (1959), 363–373.

4440:
Berencz, F. Die Rolle der Multipolwechselwirkungen bei den van der Waalsschen Attraktionen. Acta Phys. Acad. Sci. Hungar. **12**, 1–12 (1960). (Russian summary)

Author's summary: "Der Ausdruck für die Wechselwirkung zwischen zwei Molekülen wird so verallgemeinert dass die Dipol-, Quadrupol- und Oktupolwechselwirkungen gleichmässig berücksichtigt werden. Es wurde von Margenau festgestellt, dass die Dipol-Quadrupol- und Quadrupol-Quadrupol-Wechselwirkungen neben der Dipol-Dipol-Wechselwirkung bei der Untersuchung der Wechselwirkung zwischen nichpolaren Molekülen eine wichtige Rolle spielen. In dieser Arbeit wird darauf hingewiesen, dass die Dipol-Oktupol-Energie grössenordnungsmässig der Quadrupol-Quadrupol-Energie gleich ist, und so muss man in Näherungsverfahren beide entweder berücksichtigen oder ausser Betracht lassen. Es wird weiterhin festgestellt, dass in dem Ausdruck für die Störungsenergie bei der Wechselwirkung zwischen zwei Wasserstoffatomen die Dipol-Quadrupol-Energie 53%, die Quadrupol-Quadrupol-Energie 13%, die Dipol-Oktupol-Energie 23% der Dipol-Dipol-Energie beträgt."

4441:
Golden, S. On a quantum mechanical theory of absolute reaction rates. Nuovo Cimento (10) **15** (1960), supplemento, 335–373.

A formal quantum mechanical theory of chemical reaction rates is formulated with due regard to the introduction of time independent projection operators, the "species-classification" operators, which are assumed sufficient to serve to classify a collection of fundamental particles as one set of molecular species (stable or metastable) or another. This allows a straightforward construction of the dynamical variables for the numbers of

each chemical species of a macroscopic system. Expressions for the time rate of change of the ensemble averaged number of each species, resembling the usual phenomenological equations of chemical kinetics, are obtained and permit a possible identification of kinetic "rate constants". The necessary conditions under which these "rate constants" are stationary with respect to variations of the statistical matrix of the system and the "species-classification" operators correspond essentially to detailed balancing for the transition rates of the system as well as microscopic reversibility of forward and inverse transitions. The expression obtained for the stationary "rate constant" furnishes an upper bound to the rate constant under the most general conditions and may be exhibited in a form which is suggestive of the activated complex theory of chemical kinetics. Serious deficiencies of the theory, recognized by the author, concern his lack of consideration of the detailed temporal evolution of the statistical matrix necessary to assure the proper temporal behavior of his "rate constants" and his inability to prove the negative definiteness of his "rate constants" without which these entities do not have their usual physical significance. *H. L. Frisch* (Murray Hill, N.J.)

QUANTUM MECHANICS

See also A3698, A3741, 4174, 4178, 4441, 4510.

4442:

Houston, William V. ★Principles of quantum mechanics: Nonrelativistic wave mechanics with illustrative applications. Dover Publications, Inc., New York, 1959. vii+288 pp. Paperbound: $1.85.

This is a paperback reprint of a book previously published in 1951 [McGraw Hill, London]. Starting from a historical review of the events which led to the quantum hypothesis, the book presents in a clear and lucid fashion the usual formulation of quantum mechanics (state vectors, probability interpretation, observables-linear operators relation) in the Schrodinger representation. The formalism is illustrated with the treatment of the one and two dimensional harmonic oscillator and the hydrogen atom. Approximation methods are then presented, in particular the WKB method, the Rayleigh-Schrodinger theory, the variational method. The time dependent perturbation theory is briefly sketched. The outline of the general principles concludes with the presentation of the Pauli description of the spin of the electron and the quantum mechanical description of a system of many identical particles (Pauli exclusion principle).

Two long chapters are then devoted to the spectroscopy of hydrogen-like and two electron systems (including the Zeeman effect). Two shorter chapters outline the quantum theory of collision. A long chapter presents the motion of an electron in a periodic potential and the book concludes with a sketch of the quantum theory of radiation (quantization of the radiation field, interaction of an electron with quantized radiation field).

The reissuing of this book is to be welcomed, for its treatment of spectroscopy and of the motion of electrons in periodic potentials are masterly. The book is well suited for an introductory senior course in quantum mechanics. The material on spectroscopy and on electrons in metal also constitute the minimum on these subjects that any first year graduate course in quantum mechanics should include. *S. Schweber* (Waltham, Mass.)

4443:

Heine, Volker. ★Group theory in quantum mechanics: An introduction to its present usage. International Series of Monographs on Pure and Applied Mathematics, Vol. 9. Pergamon Press, New York-London-Oxford-Paris, 1960. ix+468 pp. $15.00.

It is interesting to observe that the number of books dealing with the application of group-theoretical methods in physics is steadily increasing, in spite of the fact that there are several recognized "classics" in this vein. The reason for this phenomenon is probably the widening recognition of the immense importance played by symmetry properties in Nature. The real understanding of a subject is tremendously enhanced by grasping the pattern of underlying order, often hidden by accidental features. This conviction is the main motivation of the author of the present book.

The text deals alternatively with mathematical and physical problems. The concept and possibility of application of groups is introduced by focusing attention to the symmetry properties of quantized systems. The theory of group representations is worked out along side of the quantum theory of a free atom, although much of the mathematics is relegated to the Appendix. The structure and vibrations of molecules and several problems of solid state physics are dealt with as further applications. The last two chapters (and these are the less fortunate parts of the book) are devoted to problems of nuclear physics and relativistic quantum mechanics.

The basic mathematical notions are worked out clearly and even the beginner will not have many difficulties in using the book to his best advantage. The notation, however, is sometimes obscure. An extensive bibliography, including the review of some standard works, completes the volume. *P. Roman* (Boston, Mass.)

4444:

Wightman, A. S. Relativistic invariance and quantum mechanics. Nuovo Cimento (10) **14** (1959), supplemento, 81–94.

This article (which is the content of a set of lectures given at the Varenna Summer School of 1959) summarizes our present knowledge of the consequence of relativistic invariance when imposed on the usual description of the states of a quantum mechanical system in terms of rays in Hilbert space. Starting from a discussion of the notions of bodily and subjective identity of physical system and of relativistic invariance based on the active and passive point of view, the requirement of relativistic invariance is eventually formulated in the active point of view in terms of transition probabilities. It asserts that for every physically realizable ray Φ there exists a corresponding (physically realizable) vector $T(g)\Phi$, and furthermore that this transformation $\Phi \to T(g)\Phi$ is such as to preserve physical relations between states in the sense that

$$|(\Phi, \Psi)|^2 = |(T(g)\Phi, T(g)\Psi)|^2.$$

A relativistically invariant theory thus determines a

realization of the relativity group by ray correspondences on the physically realizable rays which preserves transition probabilities. The theorems which assert that for the connected part of the Lorentz group $T(g)$ is a unitary representation up to a factor ± 1 are presented. The continuous unitary representation of the inhomogeneous Lorentz group are then briefly discussed.

The article includes brief but succinct statements on the consequences of superselection rules and a brief analysis of the representations of the Lorentz group including inversions. *S. Schweber* (Waltham, Mass.)

4445:

Ter-Martirosyan, K. A. Incompatibility of the conditions of analyticity and unitarity in the Lee model. Ž. Èksper. Teoret. Fiz. **37** (1959), 1005–1009 (Russian); translated as Soviet Physics. JETP **10** (1960), 714–717.

This paper is an attempt to investigate the properties of the Lee model without use of a Hamiltonian formalism. Instead, the author tries to use only general arguments, like analyticity properties of scattering amplitudes (causality) and unitarity. Contrary to what is the case for more realistic models the unitarity condition is rather simple here because of the special selection rules one has in the Lee model. The solution of the general equations the author writes down is not unique. One has the same phenomenon here as has been discussed by Castillejo, Dalitz and Dyson [Phys. Rev. (2) **101** (1956), 453–458] for another but similar problem. The author shows that none of the solutions he can obtain is physically admissible as all of them have the anomalous state that one has earlier found with the aid of the Hamiltonian formalism. *G. Källén* (Lund)

4446:

Królikowska, Z. Relations between the polynomial method and the factorization method. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phys. **7** (1959), 157–168. (Russian summary, unbound insert)

The author shows that the two methods, the Schrödinger factorization and the Sommerfeld polynomial methods, are almost equivalent for solving the classic quantum eigenvalue problems, with a slight advantage to the polynomial method. *L. Infeld* (Warsaw)

4447:

Ghosh, N. N. A matrix method of reduction of the general Lorentz transformation. Bull. Calcutta Math. Soc. **51** (1959), 103–108.

Author's summary: "The object of this paper is to present a new method of reduction of the general Lorentz transformation in 4-space into a pair of component plane rotations. Starting with the Minkowski representation of the general Lorentz transformation in which a complex orthogonal matrix of rank 4 is involved, it is shewn how by a matrix method this can generally be resolved into a pair of commutative orthogonal factor-matrices. A resolution by means of a pair of non-commutative factor-matrices with complex elements involving the coefficients of a unimodular spin transformation in complex space is also noted in this connection." *H. S. Ruse* (Leeds)

4448:

Garrido, L. M. Adiabatic invariants in quantum mechanics. Physica **26** (1960), 501–504.

Author's summary: "We show that the constants of motion of a system evolving under a time-independent hamiltonian H_0 are adiabatic invariants to the nth order, in certain cases, and to all orders in other cases, of the system evolving under a slow time-dependent hamiltonian $H=H_0+H_1(t)$."

4449:

Tharrats, Jésus. Sur le théorème de von Neumann concernant l'indéterminisme essentiel de la Mécanique quantique. C. R. Acad. Sci. Paris **250** (1960), 3786–3788.

Author's summary: "Nous étudions de nouveau le problème de l'impossibilité d'annuler les dispersions de toutes les grandeurs observables, en centralisant la question dans le théorème de von Neumann que nous présentons sous une forme modifiée. La solution de cet auteur nous paraît douteuse principalement du fait que la démonstration est faite pour les projecteurs, lesquels ne représentent pas des grandeurs observables."

4450:

Bunkin, F. V. On the parametric excitation of quantum systems. Radiotehn. i Èlektron. **5** (1960), 296–300 (Russian); translated as Radio Engrg. and Electronics **5**, no. 2, 168–174.

Author's summary: "The general problem of parametric regeneration in quantum systems is formulated. A concrete example of spin-system regeneration is considered."

4451:

Lyubošic, V. L. Polarization phenomena in radiative collisions of two electrons. Ž. Èksper. Teoret. Fiz. **37** (1959), 1727–1740 (Russian. English summary); translated as Soviet Physics. JETP **10** (1960), 1221–1228.

Author's summary: "Two-row matrix formalism is used for the description of polarization phenomena in the collision of two Dirac particles of arbitrary energy. Radiative collisions of two polarized electrons are considered. The structure of the general formulas for the polarization parameters after the collision is investigated, and concrete calculations are carried out in the ultrarelativistic and nonrelativistic limits taking into consideration polarization of the target electrons."

4452:

Fivel, Daniel I.; Klein, Abraham. On the analytic properties of partial wave amplitudes in Yukawa potential scattering. J. Mathematical Phys. **1** (1960), 274–279.

The author considers the Schrödinger equation for the lth partial wave $\phi_l(k, r)$ with a Yukawa potential. It is shown that the l-wave scattering amplitude satisfies a dispersion relation with cuts on the real line $(-\infty, -\mu^2)$ and $(0, \infty)$, and a finite number of poles in the interval $(-\mu^2, 0)$. The difficult step is to show analyticity in the upper half-plane (as opposed to the lower half-plane, where results for more general potentials are known) and it is here that he uses the fact that the potential is of the Yukawa type; this ensures that the kernel in the integral

equation for $f_l(k, r)$ behaves well enough at ∞ for the solution to be holomorphic in the cut-plane.

The case treated in this paper, and its obvious generalization to sums of Yukawa potentials, were covered in an anterior article [A. Martin, Nuovo Cimento (10) **14** (1959), 403–425; MR **22** #3464], but the new proof is completely rigorous. *R. F. Streater* (Princeton, N.J.)

4453:

Ohmura, Takashi. Minimum property in the Hulthén-type variational methods. J. Mathematical Phys. **1** (1960), 27–34.

The minimum property of different variational methods for evaluating the scattering length is discussed. For the S-wave equation $Lu=d^2u/dr^2+W(r)u=0$, $u(0)=0$, $0<r<\infty$, the first method of Hulthén with the auxiliary condition $\int_0^\infty u_t L u_t dr=0$ for the trial function u_t [Kungl. Fysiogr. Sällsk. i Lund Förh. **14** (1944), no. 21; MR **6**, 11] is shown to give an upper bound for the scattering length a if there is no bound state and if the trial function u_t is sufficiently accurate (for which there is however no criterion). The author further asserts that the second Hulthén as well as the Kohn method can give upper bounds for a only under certain additional assumptions which are usually unlikely to be satisfied. These conclusions are rather misleading, however, in view of a rigorous proof by Spruch and Rosenberg [Phys. Rev. (2) **116** (1959), 1034–1040; **117** (1960), 1095–1102; MR **22** #3461, #3462] and Rosenberg, Spruch and O'Malley, ibid. **118** (1960), 184–192; MR **22** #3463] that the Kohn variational expression is actually an upper bound on a in the absence of bound states. Thus the author's preference for the first Hulthén method does not seem well founded, inasmuch as it involves an ambiguous double-valued functional. Also a complex problem (electron-hydrogen scattering) is considered with similar conclusions, to which the same remark as above applies. *T. Kato* (Tokyo)

4454:

Ohmura, Takashi. Stationary expression for effective range. J. Mathematical Phys. **1** (1960), 35–40.

The phase shift $\eta=\eta(k^2)$ for the wave equation $Lu\equiv d^2u/dr^2+W(r)u+k^2u=0$, $0<r<\infty$, is defined by the asymptotic form $u\sim\text{const}\sin(kr+\eta)$, $r\to\infty$ ($W(r)$ is assumed to tend to zero rapidly for $r\to\infty$). The scattering length a and the effective range r_0 are then defined by $k\cot\eta=-a^{-1}+r_0k^2/2+o(k^2)$, $k\to0$. Several variational expressions are known for a [see, for example, the preceding review and Spruch and Rosenberg, Phys. Rev. (2) **116** (1959), 1034–1040; MR **22** #3461]. In this paper the author deduces variational expressions for r_0. First it is shown that r_0 is the stationary value of the quadratic functional $Q[w_t]\equiv r_{0t}+\frac{1}{2}\int_0^\infty w_tL_0^2w_t dr$, where $L_0=d^2/dr^2+W(r)$ and the trial function w_t should satisfy the boundary and asymptotic conditions $w_t(0)=0$, $L_0w_t(0)=0$, $w_t\sim r_{0t}r-r^2+\alpha r^3/3$, $r\to\infty$ (r_{0t} and α are constants). Another functional with the same property is $Q'[u_{0t}, w_t]=r_{0t}-\int_0^\infty(w_tL_0u_{0t}+2u_{0t}^2+u_{0t}L_0w_t)dr$, depending on two trial functions u_{0t} and w_t, with the same boundary and asymptotic conditions as above for w_t and with $u_{0t}(0)=0$, $u_{0t}\sim1-\alpha r$, $r\to\infty$ (with the same α as above). The "effective range for bound states" is also considered, with similar variational expressions. The minimum nature of the stationary values of these functionals is discussed, but the result does not seem to be conclusive. *T. Kato* (Tokyo)

4455:

Kato, T.; Kuroda, S. T. A remark on the unitarity property of the scattering operator. Nuovo Cimento (10) **14** (1959), 1102–1107. (Italian summary)

Consider a system of two particles moving in a one-dimensional space. The Hamiltonian H of the system is

$$H=-\frac{\partial^2}{\partial x_1^2}-\frac{\partial^2}{\partial x_2^2}+V(x_2)+K=H_0+V(x_2)+K,$$

where the potential $V(x_2)$ of particle 2 is such that $\int|V(x_2)|dx_2<\infty$ and $\int|V(x_2)|^2dx_2<\infty$. Furthermore it is assumed that the Hamiltonian of particle 2, $H_{12}=-d^2/dx_2^2+V(x_2)$, considered as an operator in the Hilbert space consisting of all complex-valued absolutely-integrable functions defined on E_l (E_l is the l-dimensional Euclidean space) has one and only one negative eigenvalue. The interaction potential K is assumed to be a self-adjoint integral operator whose kernel is of the form $c\varphi(x_1, x_2)\overline{\varphi(\xi_1, \xi_2)}$, with φ a fixed function in $L^2(E_2)$ and c a real number. ($Ku=c(u, \varphi)\varphi$.) It is then shown that the wave operators $W_\pm$, $W_\pm(H, H_0)=\lim_{t\to\pm\infty}e^{itH}e^{-itH_0}$ exist, but that with a suitable choice of K the scattering operator S, $S=W_+(H, H_0)^*W_-(H, H_0)$, is not unitary. It is to be noted that $H_0=\partial^2/\partial x_1^2+\partial^2/\partial x_2^2$ has no bound states whereas H does. The proof consists essentially in showing that the range $W_+(H, H_0)$ is not equal to the range of $W_-(H, H_0)$. *S. Schweber* (Waltham, Mass.)

4456:

Ioffe, B. L. Analyticity and unitarity in the scattering of scalar mesons from a static nucleon. Ž. Èksper. Teoret. Fiz. **37** (1959), 1764–1769 (Russian. English summary); translated as Soviet Physics. JETP **10** (1960), 1245–1248.

Author's summary: "The uniqueness of the determination of the scattering amplitude from the analyticity and unitarity conditions is considered for the problem of scattering of scalar neutral and charged mesons from a static nucleon in the one-meson approximation. The analysis is carried out by describing the analytic properties of the scattering amplitude on all sheets of the Riemann surface. As a result it is found that the non-unique terms appearing in the solution are characteristic of scattering from virtual or Breit-Wigner levels."

4457:

Muradyan, R. M. Azimuthal asymmetry during the scattering of Dirac particles. Dokl. Akad. Nauk SSSR **131** (1960), 1057–1059 (Russian); translated as Soviet Physics. Dokl. **5**, 352–355.

4458:

Inopin, E. V.; Tiščenko, B. I. Scattering of electrons by light nonspherical nuclei. Ž. Èksper. Teoret. Fiz. **37** (1959), 1308–1318 (Russian. English summary); translated as Soviet Physics. JETP **10** (1960), 932–938.

Authors' summary: "The scattering of electrons by

nonspherical nuclei is treated in the Born approximation. Expressions for the elastic and inelastic cross sections have been derived for the general case of oriented nuclei with arbitrary deformations. The theory is compared with the experiments on the inelastic scattering of electrons by light nuclei."

4459:

Landshoff, P. V. A discussion of dual diagrams in perturbation theory. Nuclear Phys. **20** (1960), 129–135.

Author's summary: "A discussion is given of the dimensions of the space in which dual diagrams are drawn and of restrictions arising for singularities in scattering processes in which more than one particle is produced."

4460:

Tietz, T. Phase shift analysis of combined Coulomb and nuclear scattering. Acta Phys. Acad. Sci. Hungar. **11**, 235–238 (1960). (Russian summary)

Author's summary: "In this paper we derive an exact formula for the phase shift of combined Coulomb and nuclear scattering. The formula derived for the above-mentioned phase shift allows us to calculate the phase shift exactly if only we know a sufficiently large zero of the solution of the Schrödinger equation containing Coulomb and nuclear potential. The above-mentioned zero has to be computed numerically."

4461:

Halfin, L. A. A new form of dispersion relations. Dokl. Akad. Nauk SSSR **130** (1960), 299–302 (Russian); translated as Soviet Physics. Dokl. **5**, 91–94.

As a generalization of the usual forward dispersion relations, a function $f(E)$ regular in the upper half-plane is written in terms of its real part in certain intervals $[a_k, b_k]$ and its imaginary part in the remaining intervals $[b_k, a_{k+1}]$, making use of a form of Cauchy's theorem given by M. V. Keldyš and L. I. Sedov [same Dokl. **16** (1937), 7–10]. Some auxiliary relations are obtained, which state that $f(E)$ is bounded at the end points a_k, b_k. By specializing, various forms for the dispersion relation are obtained, including those of W. Gilbert [Phys. Rev. (2) **108** (1957), 1078–1083; MR **19**, 1019].

Although the new forms do not contain more information, they may have some practical advantages over the old; they converge more rapidly at ∞, and they allow the most effective use of experimental results. For example it is possible to use data from phase shift analysis, giving $\mathrm{Re}\, f$ in the region where it is most reliably determined, and data from the total cross-sections in other regions. *R. F. Streater* (Princeton, N.J.)

4462:

Ascoli, R.; Minguzzi, A. Analyticity properties of production amplitudes. Phys. Rev. (2) **118** (1960), 1435–1438.

Authors' summary: "We study the analytic properties of production amplitudes as functions of the momentum transfer Δ^2 between one of the incoming particles and one of the outgoing particles, when the total energy and the three further parameters determining the relative motion of the three outgoing particles in the center-of-mass system are held fixed. We find that suitable combinations of the amplitudes are analytic functions of Δ^2 regular within an ellipse in the Δ^2 plane. It is also shown that in the same domain the cross section $\partial^2\sigma/\partial\Delta^2\partial w^2$ is an analytic and regular function of Δ^2, w^2 being the total mass of two of the outgoing particles. The poles in Δ^2 conjectured by Chew and Low never lie inside the domain of regularity."

4463:

Popov, V. S. On the theory of the relativistic transformations of the wave functions and density matrix of particles with spin. Ž. Èksper. Teoret. Fiz. **37** (1959), 1116–1126 (Russian); translated as Soviet Physics. JETP **10** (1960), 794–800.

The wave-functions of a particle of spin S, and of a system of two particles of spins S_1 and S_2 are given in terms of the infinite-dimensional irreducible unitary representations of the Lorentz group. An invariant classification of the states is discussed, as well as the general form of the density matrix for the two-particle case. No applications are given.

H. W. Lewis (Madison, Wis.)

4464:

Scarf, F. L. Thirring model with variable interaction. Phys. Rev. (2) **117** (1960), 868–872.

The two dimensional relativistic model of Thirring [Ann. Physics **3** (1958), 91–112; MR **19**, 1016] is modified by introducing a variable coupling constant, $\lambda = \lambda_0 f(x, t)$. The author shows that in this modified model, all renormalized quantities exist if $\lambda \to 0$ along the past light cone and the model allows production of real physical particle pairs. *A. Raychaudhuri* (Calcutta)

4465:

Capella, Alphonse. Sur la quantification du champ électromagnétique libre en Relativité restreinte. C. R. Acad. Sci. Paris **251** (1960), 636–638.

The author performs a Lorentz-invariant Fourier transformation of the electromagnetic field (which is assumed to satisfy the Maxwell equations in vacuo and to satisfy boundary conditions sufficient to guarantee the convergence of all interesting integrals, such as those of energy and linear momentum) by utilizing an invariant measure defined on the light cone. From the resulting complex amplitudes, which are functions of the (null) propagation vector and the sense of plane polarization, he forms linear combinations which may be considered the creation and annihilation operators, respectively, of photons of stated propagation and circular polarization. It is shown that energy, linear momentum, and angular momentum are the expected integrals over the bilinear products of creation and annihilation operators. All the manipulations performed are straightforward and routine.

P. G. Bergmann (Syracuse, N.Y.)

4466:

Kaempffer, F. A. On the possible existence of a derivative coupling in quantum electrodynamics. Canad. J. Phys. **37** (1959), 1339–1343.

The author suggests that the muon possesses an extra

electromagnetic interaction in the form of a derivative coupling to the electromagnetic field. It is suggested that such an interaction might account for the anomalously large mass of the muon, since more violent divergences enter in gradient coupled theories. Specifically, the new interaction is of the form $L' = -e\lambda[\bar{\psi}(x), \psi(x)]\partial_\mu A^\mu$, where λ is a new constant with the dimensions of a length. The reviewer does not see how the author proposes to make this theory gauge invariant.

R. Arnowitt (Syracuse, N.Y.)

4467:

Ohnuki, Yoshio; Takao, Yasutaro; Umezawa, Hiroomi. On the redundant solutions of the Bethe-Salpeter equation. Progr. Theoret. Phys. **23** (1960), 273–283.

If spin zero or one-half particles are considered to bind each other by a spin zero field, the ladder approximation to the Bethe-Salpeter equation possesses solutions of two types. There are normal solutions which arise also in the non-relativistic equations without relative time, and for which the binding energy vanishes in the limit of no coupling. There are also abnormal solutions which are still bound in the absence of coupling and do not appear in the non-relativistic equations. The authors show that these abnormal bound states are not eigenstates of the complete Hamiltonian, so are not admissible on physical grounds. This is also shown in detail for the static nucleon scalar meson model. It may be possible that going beyond the ladder approximation in the Bethe-Salpeter equations removes these anomalous solutions.

John G. Taylor (Paris)

4468:

Tarski, Jan. Analyticity of the fourth order scattering amplitude with two complex invariants. J. Mathematical Phys. **1** (1960), 149–163.

The discussion of perturbation diagrams in each or any order of the coupling constant is of importance in keeping conjectures of analyticity for scattering amplitudes on the path of the probable. In this paper the author considers the analyticity properties of the fourth order perturbation 'box' diagram for two particle elastic scattering. It extends the work of Karplus, Sommerfeld and Wichmann [Phys. Rev. (2) **114** (1959), 376–382; MR **21** #4024] who considered the analyticity of the diagram for real values of two of the possible invariants for fixed values of the four external masses. The author extends this discussion to complex values of these two invariants. The method he uses is an extension of that of Eden [Proc. Roy. Soc. London Ser. A **210** (1952), 388–404; MR **13**, 1011] and relates the singularities of the amplitude to singularities of the integrand without carrying out the integration explicitly. The points of the surface of singularity which are on the physical sheet are determined, and the condition on the internal and external masses for the existence of complex singularities is obtained. The behaviour of the singularities as the external masses are varied is also considered, and so is the behaviour of the amplitude near the singularities. These results will be of value in attempts to use those analyticity properties of scattering amplitudes which are still valid when the Mandelstam representation is not, due to the presence of complex singularities.

John G. Taylor (Paris)

4469:

Garrido Arilla, Luis M. a; Pascual de Sans, P. The electron of the universal Fermi interaction. An. Real Soc. Españ. Fis. Quim. Ser. A **55** (1959), 5–12. (Spanish. English summary)

A canonical formalism for the two component electron theory of Feynman [Feynman and Gell-Mann, Phys. Rev. (2) **109** (1958), 193–198; MR **19**, 813] is presented. The Lagrangian density $L = L_e + L_p + L_i$ is used, where

$$L_e = -\partial_\mu\varphi^+\partial_\mu\varphi - m^2\varphi^+\varphi,$$

$$L_p = -\tfrac{1}{2}\partial_\mu A_\lambda \cdot \partial_\mu A_\lambda,$$

$$L_i = ie\varphi^+ A_\mu \partial_\mu\varphi - ie\partial_\mu\varphi^+ A_\mu\varphi - e^2\varphi^+ A_\mu A_\mu\varphi + \tfrac{1}{2}ie\varphi^+\sigma_\mu^{-1}\sigma_\nu F_{\mu\nu}\varphi.$$

The interaction Hamiltonian obtained is

$$H_i[x, n] = -L_i(x) + e^2\varphi^+ A_\mu n_\mu A_\lambda n_\lambda\varphi - \tfrac{1}{4}e^2[\varphi^+(\sigma_\lambda^{-1}\sigma_\mu - \sigma_\mu^{-1}\sigma_\lambda)n_\lambda\varphi]^2$$

[cf. Umezawa, *Quantum field theory*, North-Holland, Amsterdam, 1956; MR **20** #690; Chapter X, § 2]. For these weak interactions discussed, parity and charge conjugation fail, where it is otherwise for time inversions.

Hing Tong (Middletown, Conn.)

4470:

Norton, R. E.; Watson, W. K. R. Commutation rules and spurious eigenstates in soluble field theories. Phys. Rev. (2) **116** (1959), 1597–1603.

The problem of the runaway solutions in electrodynamics is studied in an exactly soluble non-relativistic model in which the electron is harmonically bound to the electric dipole part of the radiation field. As in the classical electrodynamical problem, the physically unreasonable time-growing solutions can be dropped at the expense of making the theory acausal over times e^2/mc^3. The authors assert that such a microscopic acausality, provided it does not lead to macroscopic acausality, is still consistent with experiment. (The authors seem to be unaware of the paper by K. Wildermuth and K. Baumann [Nuclear Phys. **3** (1957), 612–623] which assumes the same model as they do, and covers the same ground.)

S. Bludman (Berkeley, Calif.)

4471:

Enflo, Bengt. Construction of wave equations by Eriksson's spinor formalism. Ark. Fys. **16**, 469–477 (1960).

The author investigates Lagrangians formed from two four-component spinors and shows that these lead to equations of the same type as the Dirac equation except for special choices of the parameters. For the special choices the equations are said to have no physical significance. The Lagrangians discussed involve not only hermitian forms containing the two spinors but also antisymmetric forms and are not invariant under time reversal. The choice of the Lagrangians chosen is influenced by the not yet completely published work of Eriksson of the spinor representation of the Lorentz group in six-dimensional space with four space-like and two time-like coordinates.

A. H. Taub (Urbana, Ill.)

4472:
Popovici, Andrei. Théorie conforme des champs spinoriels. I. Rev. Math. Pures Appl. **4** (1959), 577–621.

It is shown that a conformal six-dimensional space is a convenient description of both the Dirac theory and of particles of spin 1. It is claimed that non-conservation of parity follows automatically. *C. W. Kilmister* (London)

4473:
Rzewuski, J. Geometrical interpretation of conservation laws in the spinor space. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phys. **7** (1959), 571–576. (Russian summary, unbound insert)

Author's summary: "A geometrical interpretation of the conservation laws of angular momentum, isotopic spin, baryon number, charge and hypercharge is given in the framework of a spinor space introduced earlier by the author [Nuovo Cimento (10) **9** (1958), 942–949; MR **20** #5681]. Some rules restricting the possible representations to those which correspond to observed particles are discussed. It is shown that, with certain natural assumptions, the spinor space is the only one which may be used for geometrical interpretation of the whole set of known conservation laws." *A. Salam* (London)

4474:
Zaikov, Raško. Über die Spinortransformationen. Izv. Bŭlgar. Akad. Nauk. Otd. Fiz.-Mat. Tehn. Nauk. Ser. Fiz. **7** (1959), 239–259. (Bulgarian. Russian and German summaries)

4475:
Green, H. S. Normalization and interpretation of Feynman amplitudes. Nuovo Cimento (10) **15** (1960), 416–433. (Italian summary)

Author's summary: "A general method is given for normalizing Feynman amplitudes and using them to calculate expectation values. The method is easily applicable to bound states and composite states. Two applications are considered in detail. The first example is the normalization of Bethe-Salpeter amplitudes, where it is found that previously suggested normalization conditions are inadequate. The second example is the renormalization of the electron amplitude for first-order self-energy processes, and here a method is suggested for making the self-energy finite. The self-energy is three times the energy of the bare electron, in this approximation."
A. Salam (London)

4476:
Gammel, J. L.; Thaler, R. M. Phenomenology of the nucleon nucleon interaction. Progress in elementary particle and cosmic ray physics, Vol. 5, pp. 97–203. North-Holland, Amsterdam; Interscience, New York; 1960.

Since nuclear physics took on its modern aspect, one of the central problems has been to discover the nature of the forces which bind the neutrons and protons in nuclei. Progress in this field went ahead on two lines: the "fundamental" meson-theoretic and the "phenomenological" approach. The present work summarizes in detail and with great clarity the results of the phenomenological researches. Some of the topics treated are: General phenomenological considerations, analysis of p-p data, potential models, n-p data, potentials for high energy n-p scattering, radiative processes, relation to the many-body problem and to meson theory. Some hitherto unpublished material is also included.
P. Roman (Boston, Mass.)

4477:
Moshinsky, Marcos. Transformation brackets for harmonic oscillator functions. Nuclear Phys. **13** (1959), 104–116.

The author defines transformation brackets connecting the wave functions for two particles in a harmonic oscillator potential, given in terms of the particle coordinates, with the wave functions given in terms of the relative and centre of mass coordinates. The matrix elements of potentials of various kinds of nuclear forces are expressed by means of these brackets in terms of Talmi integrals [I. Talmi, Helv. Phys. Acta **25** (1952), 185–234]. Recurrence relations and expressions are given for these brackets. *N. Rosen* (Haifa)

4478:
Solov'ev, V. G. Parity correlations in light nuclei. Dokl. Akad. Nauk SSSR **131** (1960), 286–289 (Russian); translated as Soviet Physics. Dokl. **5**, 298–300.

4479:
Fonda, Luciano; Newton, Roger G. Theory of resonance reactions. Ann. Physics **10** (1960), 490–515.

Authors' summary: "A general formal theory of resonance reactions and scattering is developed without the use of channel radii. The approach employed allows a simple physical interpretation of the two energies, one of which must be kept fixed while the other one is varied in order for a Breit-Wigner denominator to vanish. A new result, obtained without a weak channel coupling hypothesis, is that a sharp resonance may be caused by forces which, if slightly different, would lead to a stable bound state even in the presence of strong coupling to open channels. The possibility of shapes other than the usual Breit-Wigner type is also discussed. A special formula is derived for resonances near a threshold caused by bound states just below that threshold in the same channels where the peak is seen."

4480:
Solov'ev, V. G.; Ten Gyn. On an application of the statistical variational principle to the theory of the atomic nucleus. Acta Phys. Acad. Sci. Hungar. **11**, 277–283 (1960). (Russian. English summary)

Authors' summary: "The statistical variational principle is used for investigating the superfluid state of the finite nucleus at non-zero temperature. The temperature of the phase transition of a nucleus from the superfluid to the normal state is obtained; behaviour of thermodynamical quantities at temperatures close to zero as well as close to the critical temperature is considered."

4481:
Lüders, Gerhart. Trägheitsmomente nach Inglis und das Bohr-van-Leeuwensche Theorem. Z. Naturforschg. **15a** (1960), 371–377. (English summary)

Author's summary: "It has been stated by Bohr and Mottelson that Inglis' method for the theoretical determination of moments of inertia of deformed nuclei, in the limit of a great number of non-interacting particles leads to the moment of inertia of rigid rotation. Recently doubts have been raised regarding the general validity of this statement. In the present paper the proof of the assertion is given in detail and its relation to the Bohr-van-Leeuwen theorem is discussed."

4482:
Jauho, Pekka. Subcritical assemblies with spontaneous fission source. Ann. Acad. Sci. Fenn. Ser. A. VI. No. 41 (1960), 16 pp.

4483:
Rapoport, L. P.; Kadmenskiĭ, S. G. Distribution of density of protons with given angular momentum in the nucleus. Ž. Èksper. Teoret. Fiz. **37** (1959), 1303–1307 (Russian. English summary); translated as Soviet Physics. JETP **10** (1960), 929–931.

Authors' summary: "The density of protons of given angular momentum is computed from the experimental distribution of the total proton density. Spatial separation of nuclear shells is demonstrated using the distribution thus obtained."

4484:
Van de Vorst, Albert. Sur la validité du champ de forces centrales dans les vibrations moléculaires. Acad. Roy. Belg. Bull. Cl. Sci. (5) **45** (1959), 966–973.

4485:
Lefebvre, Roland. Une méthode de détermination des orbitales moléculaires self-consistentes. Cahiers de Phys. **13** (1959), 369–428.

Equations determining self-consistent molecular orbitals are obtained from the condition that the first order variation of the total energy should be zero. Starting from an approximate solution of these equations (nearly self-consistent orbitals) a more accurate solution is obtained by expanding the equations in terms of the changes in the coefficients of the orbitals and retaining all terms of the first order in these changes. An iteration of this process leads to accurately self-consistent molecular orbitals. This procedure has two important advantages. It is much more rapidly convergent than the usual iterative solution of the equations of Roothaan [Rev. Mod. Phys. **23** (1951), 69–89] and it is immediately applicable to configurations with unclosed shells. An alternative derivation of the basic equations of the new method is obtained from a configuration interaction calculation with subsidiary conditions. These conditions ensure that (for the closed shell case) the total wave function is expressible as a single Slater determinant.

Various applications of the theory are given. These include a proof of the property of alternation for conjugated hydrocarbon radicals and a perturbation method for molecular orbital theory based on a single iteration of the basic equation. It is in the latter application that the rapidity of convergence of the new method is of especial importance. *A. C. Hurley* (Melbourne)

4486:
Roman, Paul. ★Theory of elementary particles. North-Holland Publishing Co., Amsterdam; Interscience Publishers Inc., New York; 1960. xii+575 pp. $12.00.

This book is concerned almost wholly with invariance aspects of elementary particle theory. The first chapter deals with the four-dimensional orthogonal group (91 pages); the second (essentially) with Dirac algebra (30 pages); the fourth with space-reflection, time reflection and charge-conjugation transformations (165 pages); and the fifth deals with isobaric space and other possible classification-symmetries of elementary particles (147 pages). There is in addition a brief—and so far as the rest of the book is concerned, mainly irrelevant—chapter on field quantisation (Chapter III, 39 pages).

The first two chapters are mathematical and though they contain what is now standard material (standard in the sense that it can be found in other texts), the reviewer found them pleasurable reading for the excellence of the presentation and choice of topics.

Unfortunately the same cannot be said of the last two chapters. The tone and the style here become inelegant and needlessly prolix. The author does not make up his mind whether he is writing a book on, for example, discrete transformations with illustrative physical examples or a text in physics covering, again as an example, the entire subject of β-radioactivity. The chapter on classification-schemes of elementary particles is of course bound to be weak for it deals with much that is only speculative in character.

Notwithstanding this criticism let it be said clearly that the reviewer knows of no other book which comprises so many of the ideas currently occupying centre of research interest and would like to recommend it to students of the subject. *A. Salam* (London)

4487:
Destouches, Jean-Louis. La fusion en théorie fonctionnelle. C. R. Acad. Sci. Paris **250** (1960), 3596–3598.

Author's summary: "Définition d'une partie 'fondue' d'un système de corpuscules. Propriétés des ondes de parties fondues. Forme des équations."

4488:
Destouches-Aeschlimann, Florence. La fusion des corpuscules en théorie fonctionnelle. C. R. Acad. Sci. Paris **250** (1960), 3593–3595.

Author's summary: "Étude d'un système de deux corpuscules à spin en théorie fonctionnelle relativiste. En certains cas un tel système a des états à ondes relatives constantes. Deux corpuscules de spin $\frac{1}{2}$ forment ainsi un système fondu se comportant comme un corpuscule de spin 1. De là une justification physique du procédé formel de fusion."

4489:

Barašenkov, V. S. Concerning some possibilities of formulation of a relativistically invariant theory of extended particles. Ž. Èksper. Teoret. Fiz. **32** (1957), 566–569 (Russian. English summary); translated as Soviet Physics. JETP **5** (1957), 470–472.

Author's summary: "Various problems pertaining to the theory of extended particles are examined in the case of the classical (non-quantum) theory of interacting dynamically undeformable extended particles. Although the structure of the theory is not of the hamiltonian type, a formulation in lagrangian variables x_k, $\dot{x}_k$; A_μ, $\dot{A}_\mu$ as well as hamiltonian variables x_k, p_k; A_μ, π_μ can be given. The possibility of a many-time formalism is examined and the latter is found to be non-unique. However, in none of the cases was a transition to a single-time theory found, and the set of equations of motion was found to be inconsistent."

4490:

Rohrlich, F. Self-energy and stability of the classical electron. Amer. J. Phys. **28** (1960), 639–643.

Author's summary: "The classical theory of the electron, as proposed by Abraham and Lorentz, is usually presented as beset by the difficulty that the momentum and velocity of its Coulomb field are incorrectly related kinematically: $p=\frac{4}{3}m_e v$, where m_e is the electromagnetic mass defined by the electromagnetic self-energy. This problem also persists in the relativistic theory. It is shown here that the difficulty is eliminated from the relativistic theory by treating the integrals over the electromagnetic field in a relativistic fashion, i.e., taking note of their dependence on the motion of the electron. The surface dependence of the integrals representing the electromagnetic momentum and energy of the particle is essential and occurs whenever the matter tensor is not introduced. The nonrelativistic limit of this formulation then also leads to the correct relationship $p=m_e v$. The corrected Abraham-Lorentz theory still contains the stability problem, but this problem is no longer related to the transformation properties. It can be removed by renormalization."

4491:

Bočvar, D. A.; Gambaryan, N. P. A measure of the delocalization of an electron in an atomic or molecular system. Dokl. Akad. Nauk SSSR **131** (1960), 532–534 (Russian); translated as Soviet Physics. Dokl. **5**, 307–309.

4492:

Granovskiĭ, Ya. I. The electromagnetic interaction in the Heisenberg theory. Ž. Èksper. Teoret. Fiz. **37** (1959), 442–451 (Russian); translated as Soviet Physics. JETP **10** (1960), 314–320.

The Heisenberg theory of a self-interacting spinor field is extended to more general interaction forms than scalar. In the end, the scalar interaction is chosen, and it is claimed that a more careful treatment of the approximations leads to a value of the fine structure constant $\alpha=1/138.1$, compared to the experimental value 1/137.

H. W. Lewis (Madison, Wis.)

4493:

Souriau, Jean-Marie. Théorie algébrique des mésons et baryons. C. R. Acad. Sci. Paris **250** (1960), 2807–2809.

Author's summary: "On propose un isoespace à sept dimensions, dont le méson est un vecteur et le baryon un spineur; on en déduit un groupe à 21 paramètres qui conserve les interactions fortes; des lagrangiens sont proposés pour les interactions fortes et électromagnétiques."

4494:

Brueckner, K. A.; Soda, Toshio; Anderson, Philip W.; Morel, Pierre. Level structure of nuclear matter and liquid He^3. Phys. Rev. (2) **118** (1960), 1442–1446.

The authors follow the Bardeen-Cooper-Schreiffer theory, but they replace the two body potential by an effective interaction (the K-matrix).

In liquid He^3, the K-matrix is repulsive at the Fermi surface for $l=0$ and $l=1$, so that there is no modification of the level structure for these angular momentum states. For $l=2$, the K-matrix is attractive at the Fermi surface, so there is a modification of the level structure but no energy gap. The absence of a gap for $l\neq 0$ arises from the form of the single particle energy E_k:

$$(1) \qquad E_k = [\varepsilon_k{}^2+\varepsilon_0{}^2|Y_l{}^m(\mathbf{k},\mathbf{k}_0)|^2]^{1/2};$$

ε_k are the unperturbed single particle energies measured relative to the Fermi-surface, ε_0 an energy related to the strength of the K-matrix at the Fermi-surface, and $\mathbf{k}_0$ an arbitrary unit vector. There is no gap because $E_k=\varepsilon_k$ when $Y_l{}^m(\mathbf{k},\mathbf{k}_0)=0$.

The meaning of $\mathbf{k}_0$ is the following. The authors say that the liquid will break into cells with a correlation distance of about 400Å. Each cell has its own $\mathbf{k}_0$; the set of $\mathbf{k}_0$ is randomly oriented, however.

Different values of m refer to different modes of excitation. At finite temperatures the different modes should be very much mixed.

The perturbation of the level structure expected for $l=2$ should result in observable thermodynamic effects. It is not possible to evaluate the theory with any confidence in the numbers, due least of all to mixing of the m-modes, but there should be a discontinuity in the specific heat at some low temperature (about 0.1°K, but maybe 0.02°K).

In nuclear matter, the ${}^3S_1$ K-matrix is attractive at the Fermi surface. This results in a superconducting type n–p pairing. The energy gap is calculated by the authors to be 0.19 Mev, and this result is said to agree with estimates from nuclear level structure analyses. Whether the calculation actually applies to finite nuclei is stated by the authors to be debatable.

J. L. Gammel (Los Alamos, N.M.)

4495:

Sewell, G. L. The Meissner effect, gauge invariance and electron correlations. Proc. Phys. Soc. **74** (1959), 340–349.

This is a very interesting coverage of the Bardeen, Cooper and Schrieffer-Bogoliubov treatment of superconductivity as it is applied to the Meissner effect in order to determine if it can give the long-range correlation required for the Meissner effect. The method is exact and the gauge invariant presented in infinite order. However,

the method does not reveal the long-range correlation in any finite order of the perturbation. The possibility of getting satisfactory correlation is suggested.

T. Sasakawa (Cambridge, Mass.)

4496:

Parmenter, R. H. High-current superconductivity. Phys. Rev. (2) **116** (1959), 1390–1399.

The Bardeen, Cooper and Schrieffer theory of superconductivity is applied to a system in which the electrons have a high drift velocity. The effective electron-electron interaction is modified by the Doppler shift of the phonon frequencies. The reduction of the energies of some intermediate phonons results in a corresponding increase in some matrix elements of the effective interaction. This leads to the possibility of a superconducting state in a metal which is not normally superconducting, if the drift velocity is comparable with the velocity of sound. It is suggested that this phenomenon might occur even at room temperature, and some experiments designed to detect it are proposed. The question of whether it is legitimate to replace the electron-phonon interaction by an effective electron-electron interaction in this region is not considered. *D. J. Thouless* (Birmingham)

RELATIVITY

See also 4314, 4315, 4363, 4444, 4464, 4472, 4520, 4521, 4522.

4497:

Boas, Mary L. The clock paradox. Science **130** (1959), 1471–1472.

The author shows that Macduffee's proof that a geodesic in Minkowski space is a path of maximum length can be simplified. The simplification turns on the fact that there is no loss of generality in choosing an inertial system whose time-axis is parallel to the geodesic.

G. J. Whitrow (London)

4498:

Builder, Geoffrey. The resolution of the clock paradox. Philos. Sci. **26** (1960), 135–144.

This is essentially a restatement of Builder's point of view concerning the clock paradox [see Austral. J. Phys. **10** (1957), 246–262; Amer. J. Phys. **27** (1959), 656–658; MR **19**, 813; **22** #597] and its treatment by previous authors. He claims that the relative retardation of one clock with respect to the other between successive coincidences can be completely discussed in terms of special relativity and the assumption that the 'rate' of a clock depends only on its velocity and not on its acceleration. General relativity adds nothing significant to this analysis. *G. J. Whitrow* (London)

4499:

Prokhovnik, S. J. The measurement of time in special relativity. J. Proc. Roy. Soc. New South Wales **93** (1959/60), 141–147.

The author proposes that Einstein's definition of the time of reflection according to A of a light-signal leaving A at time t_1 and returning at time t_2 as the arithmetic mean of t_1 and t_2 be abandoned. Instead, he proposes a new definition which implies that A assigns different lengths to the outward and return journeys. This definition reduces to the geometric mean of t_1 and t_2. He then finds that all inertial systems keep the same time, and there is no time-dilatation. The author tries to dispose of the experimental evidence of the latter, but his views are contested by N. W. Taylor in the subsequent discussion. To the reviewer it seem that, since the author's argument implies that the observer does not regard himself to be at rest, it conflicts with the principles of relativity theory.

G. J. Whitrow (London)

4500:

Macduffee, C. C. Arc lengths in special relativity. Proc. Cambridge Philos. Soc. **56** (1960), 176–181.

The author examines the clock paradox by considering arc lengths in Minkowski space-time, first in the two-dimensional case (one spatial dimension) and then in the four-dimensional case. He shows rigorously that in Minkowski space the extremals (geodesics) represent relative maxima. He also considers separately the case when the geodesic is a light-line and proves that there is no other admissible arc connecting two points on such a line. *G. J. Whitrow* (London)

4501:

Maehara, Shôji; Tsunoda, Shukichi. Bemerkungen über die Einführung der Lorentz-Transformationen in die spezielle Relativitätstheorie. Mem. School Sci. Engrg. Waseda Univ. Tokyo. No. 22 (1958), 57–66.

4502:

Weinstein, Roy. Observation of length by a single observer. Amer. J. Phys. **28** (1960), 607–610.

4503:

Holton, Gerald. On the origins of the special theory of relativity. Amer. J. Phys. **28** (1960), 627–636.

Author's summary: "Einstein's early work on relativity theory is found to be related to his other work at that time (e.g., in subject matter and style). In addition to this element of internal continuity one finds also—as a key to a new evaluation of the significance of Einstein's contribution—an external continuity with the classic, Newtonian tradition governing restrictions on permissible hypotheses. On the other hand, Einstein's work is shown to have been, in important respects, more independent of other contemporary work in this field than has recently been proposed.

"These continuities and discontinuities are set forth to make the point that philosophical studies of scientific work should proceed on historically valid ground. Some guiding principles are indicated for dealing with conflicting source materials for such studies."

4504:

Fletcher, John George. Local conservation laws in generally covariant theories. Rev. Mod. Phys. **32** (1960), 65–87.

This is a general review and critique of the meaning

and uses of conservation laws, particularly in general relativity, in contrast to special relativistic theory. The various possible differential (local) conservation laws following from general coordinate invariance are derived and shown to be strong, i.e., to have the form of identities as functions of the field variables at a point. It is argued that just because of this fact, the conservation laws have far less interest in general relativity than do their counterparts in special relativity. From the requirement that an energy expression (as a surface integral at spatial infinity) must yield the mass for the outer Schwarzschild metric, a general form for such an integral is derived, and its invariance under asymptotically vanishing coordinate transformations discussed.

It is concluded, however, that no unique satisfactory expression for energy having all the properties associated with that concept in special relativity exists. In this connection, the reviewer wishes to remark that this problem (and others left open by the author) is believed to be satisfactorily solved in the affirmative in a recent paper by R. Arnowitt, S. Deser and C. Misner [cf. #4505b].

Finally, some mathematical uses of conservation laws are presented. These deal with (1) a simplified derivation of the Dirac field's stress tensor, (2) the relation between gauges and constraints, and (3) the treatment of gauge in the classical form of the Schwinger action principle. For the two latter items the reviewer must point out that, contrary to the author's implication, a satisfactory and direct application of the action principle and gauge-constraint relations in fact leads to a complete, explicit, expression of the dynamics of the gravitational field entirely analogous to that of any other Lorentz-invariant field. In particular, a unique energy is thereby defined in terms of the two degrees of freedom of the field.

S. Deser (Waltham, Mass.)

4505a:

Arnowitt, R.; Deser, S.; Misner, C. W. Dynamical structure and definition of energy in general relativity. Phys. Rev. (2) **116** (1959), 1322–1330.

4505b:

Arnowitt, R.; Deser, S.; Misner, C. W. Canonical variables for general relativity. Phys. Rev. (2) **117** (1960), 1595–1602.

In paper I of this series [R. Arnowitt and S. Deser, Phys. Rev. (2) **113** (1959), 745–750; MR **20** #7536], two of the authors showed how to define radiation gauge in the linearized theory of gravitation. Now they turn their attention to the full Einstein theory and discuss two related problems: (i) how to disentangle the true dynamic variables of the gravitational field from the largely arbitrary geometric variables associated with general coordinates; (ii) how to define the total energy and momentum of the field and its sources for a system whose geometry becomes flat sufficiently rapidly at large spatial distances. (In a generally covariant theory the generators of infinitesimal translations of x^μ vanish, since the coordinates are merely arbitrary parameters; one has to associate energy and momentum with translations of "invariant coordinates", four functionals of $g_{\mu\nu}(x)$ which become orthonormal at spatial infinity and are arbitrary only to the extent of an inhomogeneous Lorentz transformation. If these are chosen appropriately, four more functionals are determined by the sources together with the remaining two independent functionals, which serve as true dynamic variables.)

The solution proposed here may be described as follows. Take a one parameter family of non-intersecting spacelike hypersurfaces which become flat and parallel at spatial infinity, and introduce coordinates x^μ which become orthonormal at spatial infinity and are such that $x^0 =$ const on each hypersurface. Let g_{ij} and K_{ij} be the first and second fundamental tensors respectively of a hypersurface. Define variables conjugate to g_{ij}:

$$\pi^{ij} = -({}^3g)^{1/2}(K^{ij} - g^{ij}K),$$

in which indices have been raised with g^{ij}, the inverse of g_{ij}. Introduce an orthogonal decomposition of g_{ij} and π^{ij} according to the scheme

$$f_{ij} = f_{ij}{}^{TT} + \tfrac{1}{3}(\delta_{ij}f^T - (\nabla^2)^{-1}f^T{}_{,ij}) + \tfrac{1}{2}(f_{i,j} + f_{j,i}),$$

where $f_{ij}{}^{TT}$ is transverse ($f_{ij}{}^{TT}{}_{,j} = 0$) and traceless ($f_{ii}{}^{TT} = 0$) and ∇^2 is the flat-space Laplacian. The ten Einstein equations (regarded as relations between $\partial_0\pi^{ij}$ and g_{ij}, π^{ij}) break up into two sets: four are constraints, containing g_{ij} and π^{ij} only, which may be used to determine g^T and π^i as functionals of the remaining eight variables and of the sources; the other six contain $\partial_0\pi^{ij}$ and, as well as g_{ij} and π^{ij}, the remaining four components $g_{\mu 0}$ of $g_{\mu\nu}$. In addition one has six equations relating $\partial_0 g_{ij}$ to g_{ij} and π^{ij}, which also contain $g_{\mu 0}$. The structure of these sixteen equations is such as to leave π^T and g_i completely arbitrary: choosing them to be definite functions of x^μ amounts to imposing coordinate conditions. The authors take as their conditions

$$-\tfrac{1}{3}(\nabla^2)^{-1}\pi^T = x^0, \quad g_i = x^i.$$

These equations together with the constraints reduce the original twelve variables g_{ij}, π^{ij} to four, the canonically conjugate pairs $g_{ij}{}^{TT}$, $\pi^{ij}{}^{TT}$, which are the true dynamic variables of the radiation field. The twelve dynamic equations (those involving $\partial_0 g_{ij}$, $\partial_0\pi^{ij}$) now split up into three sets: four equations determine $g_{\mu 0}$ as functionals of $g_{ij}{}^{TT}$, $\pi^{ij}{}^{TT}$ and the sources; four remain as true equations of motion of the radiation field; four become Bianchi identities. Finally, the generator of infinitesimal variations of the field variables may be written (after discarding surface terms and total variations)

$$G = \int d^3x \pi^{ij}\delta g_{ij}$$

$$= \int d^3x(\pi^{ij}{}^{TT}\delta g_{ij}{}^{TT} + g^T{}_{,ij}\delta x^0 - (\pi^i{}_{,ij} + \pi^j{}_{,ij})\delta x^i),$$

so $-g^T{}_{,ij}$ and $-(\pi^i{}_{,ij} + \pi^j{}_{,ij})$, when expressed in terms of the canonical variables and the sources, are respectively the densities of energy and momentum of the system.

This definition of radiation gauge is not the only possible one. Any array $({}^3g)^\lambda g_{ij}$ (with $\lambda \neq -\frac{1}{3}$) or $({}^3g)^\lambda g^{ij}$ (with $\lambda \neq \frac{1}{3}$) and a corresponding conjugate array constructed from g_{ij} and K_{ij} will do as a starting point for an orthogonal decomposition of the type described.

P. W. Higgs (Edinburgh)

4506:

Gutman, I. I. A generally covariant method of successive approximations in general relativity. Ž. Èksper. Teoret. Fiz. **37** (1959), 1639–1645 (Russian. English

summary); translated as Soviet Physics. JETP **10** (1960), 1162–1166.

The author proposes an approximation method for gravitational equations based on the development with respect to k/c^2, where k is the gravitational constant and c the velocity of light. *L. Infeld* (Warsaw)

4507:

Das, Anadijiban. Spinning charged test-particles in general relativity. Progr. Theoret. Phys. **23** (1960), 610–615.

The general relativistic equations of motion for charged test particles in electromagnetic or vector-meson fields are derived. The derivation is based on the Fock-Papapetrou method. An expansion procedure with respect to the parameter δx^i (the deviation from the world line of the particle) is used. In the third order of approximation one gets the equations for charged test particles. The next step of this approximation method leads to the equations of motion of spinning charged test particles.

S. Bażański (Warsaw)

4508:

Popovici, Andrei. Théorie conforme relativiste générale des champs tensoriels et spinoriels. II. Les champs spinoriels. Com. Acad. R. P. Romîne **9** (1959), 671–677. (Romanian. Russian and French summaries)

[For part I see same Com. **8** (1958), 877–886; MR **21** #4040.]

4509:

Magnusson, M. Further properties of the energy-momentum complex in general relativity. Mat.-Fys. Medd. Danske Vid. Selsk. **32**, no. 6, 22 pp. (1960).

This paper represents an extension of C. Møller's work in Ann. Physics **4** (1958), 347–371 [MR **20** #732]. Møller had shown that a certain expression for the energy-momentum complex of general relativity introduced by him is uniquely determined by the requirement that (a) T_i^k is an affine tensor density of weight 1, (b) T_4^μ ($\mu = 1, \cdots, 4$) are a scalar density and the components of a 3-vector density, respectively, under purely spatial coordinate transformations, (c) the v. Freud superpotential associated with the complex depends on no higher than first-order derivatives of the metric tensor. In the present paper it is shown that the last of these three requirements is unnecessary. It is also shown that all components of the energy complex can be made to vanish along a geodesic (not merely at one world point) by a suitable choice of coordinate system. [Note by reviewer: cf. also A. Komar, Phys. Rev. (2) **113** (1959), 934–936; MR **21** #1196.]

P. G. Bergmann (Syracuse, N.Y.)

4510:

Peres, Asher; Rosen, Nathan. Quantum limitations on the measurement of gravitational fields. Phys. Rev. (2) **118** (1960), 335–336.

It is argued that the gravitational field needs to be quantized because the approximate weak quasi-static field is subject to uncertainty relations. This is proved by observing that the equation of motion for slow motions is analogous to that of a particle in an electromagnetic field, and using the analogue of the Bohr-Rosenfeld discussion. *C. W. Kilmister* (London)

4511:

Tits, J. Les espaces isotropes de la relativité. Colloque sur la théorie de la relativité 1959, pp. 107–119. Centre Belge Rech. Math., 1960.

Synopsis de l'auteur: "Nous nous proposons de donner ici une énumération des divers espaces homogènes et 1-isotropes (donc, en particulier, des espaces homogènes et e-isotropes) existants, en traitant successivement le problème local et le problème global. Les démonstrations détaillées des résultats exposés paraîtront ailleurs; nous nous bornerons ici à en indiquer le principe."

Il s'agit ici des espaces localement [globalement] homogènes et localement [globalement] 1-isotropes, et e-isotropes. *V. Hlavatý* (Bloomington, Ind.)

4512:

Bertotti, Bruno. Structure of the electromagnetic field. Phys. Rev. (2) **115** (1959), 742–745.

This paper is an investigation of certain properties of the "already unified" field theory of Misner and Wheeler [Ann. Physics **2** (1957), 525–603; MR **19**, 1237] in which the electromagnetic field is to be obtained purely from the metric. Three items are examined: (a) The algebraic conditions of the theory are re-expressed in terms of eigenvalues of the Ricci tensor R_μ^ν. The conditions are thus put in a form in which they are independent of each other. (b) The fourth order condition is given a geometrical interpretation in terms of the blades of the Ricci tensor. (c) The null electromagnetic field is examined. It is shown in this case that the field cannot be recovered from the Ricci tensor, except in the case where the field is not everywhere null in the neighborhood of the point under consideration. *R. Arnowitt* (Syracuse, N.Y.)

4513:

Graef-Fernández, Carlos. Orbits in Birkhoff's central field. Proc. Sympos. Appl. Math., Vol. 9, pp. 167–189. American Mathematical Society, Providence, R.I., 1959.

Birkhoff's theory is used to study the motion of a particle in the gravitational field of a mass point at rest in an inertial coordinate system. The orbits are planar. Radial and circular orbits are discussed first. A generalization of the method of the potential well is used to classify the remaining orbits. Besides orbits corresponding in some manner to the classical Newtonian orbits there are orbits which wind asymptotically towards circles from the inside or outside and curved orbits which are captured by the central mass point. Approximate calculations are made for the solar system of the advance of perihelion for "elliptic" orbits. The behaviour of a photon in the solar system is also analyzed. The results, within the approximations used, agree with Einstein's theory. The stability of circular orbits is examined. For a treatment of the same problem in the Einstein theory, see the article of Darwin, Proc. Roy. Soc. London. Ser. A **249** (1959), 180–194 [MR **20** #5671]. *M. Weisfeld* (Menlo Park, Calif.)

ASTRONOMY

See also 4398, 4551.

4514a:

Yarov-Yarovoĭ, M. S. Recurrence relations for calculation of the derivatives of compound functions. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1958**, no. 6, 31–37. (Russian)

4514b:

Yarov-Yarovoĭ, M. S. Initial terms of the expansion of the force function of the attraction of two bodies. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1958**, no. 6, 39–44. (Russian)

4514c:

Yarov-Yarovoĭ, M. S. On expansion of the force function of the attraction of two bodies in the moments of inertia for non-principal and non-central axes of inertia. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 3, 55–62. (Russian)

4514d:

Yarov-Yarovoĭ, M. S. The expansion of the force function of the attraction of two bodies under some assumptions on the distribution of their mass. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 6, 58–63. (Russian)

These papers concern the technique of computing the gravitational potential of two bodies of general mass distribution. In the first paper a recursion formula is established to simplify application of the chain rule of calculus for functions of several variables. In the second a series expansion of the potential is given in terms of moments of the mass distributions; a number of simplifications are made and a large number of terms are given explicitly. In the third paper a new form of the expansion is given, based on inverse powers of the distance between two reference points in the respective bodies. In the fourth paper this last series is simplified for certain special cases: for example, one body is taken to be a particle, or the mass distributions are assumed to have symmetry properties. *W. Kaplan* (Ann Arbor, Mich.)

4515:

Butkevič, A. V. The application of geodetic principles to practical astronomy. Astr. Ž. **37** (1960), 161–172 (Russian. English summary); translated as Soviet Astr. AJ **4**, 154–165.

Three problems are discussed: (1) the application of the theorem of Legendre, on the calculation of small spheroidal triangles (as if they were plane triangles, by subtracting $\frac{1}{3}$ the spherical excess from each of the angles) to the calculation of the relative position of the North Celestial Pole and the North Star (less than 1° away), together with a study of the approximation made; (2) the calculation of chronometer corrections and their errors by the general method of the weighted arithmetic mean, taking the weights as inversely proportional to the time-intervals; (3) the calibration of an optical micrometer (value of one revolution of the screw, and the errors of the screw) by methods used for the adjustment of observations in triangulation, including both the method of angles, and the method of directions, in which the corrections are considered as belonging on the directions of rays which enclose angles.

J. A. O'Keefe (Chevy Chase, Md.)

4516:

Colombo, Giuseppe. Sui satelliti del sistema Terra-luna. Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **28** (1960), 169–172.

4517:

Warzée, J. Distance et pouvoir absorbant d'une nébuleuse obscure. Acad. Roy. Belg. Bull. Cl. Sci. (5) **45** (1959), 1063–1077.

Author's summary: "Exposé d'une méthode assez simple permettant de déduire, du diagramme différentiel de Wolf, la distance et le pouvoir absorbant d'une nébuleuse obscure fonctionnant comme un écran.

"On admet que les étoiles sont réparties suivant la fonction générale des luminosités de van Rhijn et que les densités stellaires dans l'espace obéissent à la loi de Seeliger, c'est-à-dire que les log A(m) dans la région non obscurcie peuvent être représentés par une droite.

"Deux tables facilitent l'emploi de la méthode, qui est appliquée à un exemple numérique."

4518:

Alfvén, H. Cosmical electrodynamics. Amer. J. Phys. **28** (1960), 613–618.

Author's summary: "A review is given of the developments in the field of cosmical electrodynamics. It is mentioned that the great interest in thermonuclear research has produced a considerable progress in plasma physics. This is of astrophysical interest because it is now possible to check the theories of a plasma by experiment.

"As an example, a recent experiment in a 'homopolar' machine is discussed, and its importance to the theory of the origin of the solar system is emphasized. Conclusions about the origin of the solar system are drawn. In particular, the mechanism by which Saturn's ring has been produced is discussed. It is further pointed out that the moon probably was born as a planet later captured by the earth."

4519:

Gusev, V. D. The correlation method of investigating large inhomogeneities of the ionosphere. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 6, 87–98. (Russian)

4520:

Gilbert, C. Gravitation and the principle of stationary action. Monthly Not. Roy. Astr. Soc. **120** (1960), 367–386.

A quadratic action principle is used to derive equations for pure gravitational fields. Matter is described by solutions of the field equations which are free from singularities and have a non-vanishing action density. A cosmological model is determined by this type of "matter-field", but the discrete nature of matter in the universe is taken

into account by replacing parts of the matter-field by mass particles of equal action and by free-space solutions of the field equations. Inertial systems are defined by the matter-field and the inertial mass of a particle is defined from the form of its action. The inertial mass is not constant, but varies as $(\text{epoch})^{1/2}$. The properties of the model are shown to agree with results found by Dirac [Proc. Roy. Soc. London Ser. A **165** (1938), 199–208] for a cosmological model. *D. W. Sciama* (Ithaca, N.Y.)

4521:
Hönl, H.; Dehnen, H. Allgemein-relativistische Überlegungen zur Kosmologie. Z. Physik **156** (1959), 382–398.

Part A is a rediscussion of homogeneous isotropic cosmological models in the light of new observational data reported by Sandage [Astrophys. J. **127** (1958), 513–526]; part B, a discussion of a closed static world model with two antipodal condensations, and its relevance to Mach's principle. *F. A. E. Pirani* (London)

4522:
Beauregard, Olivier Costa de. Sur l'interaction gravitationnelle au niveau quantique. C. R. Acad. Sci. Paris **250** (1960), 2521–2523.

A Kemmer type equation for gravitation is used to exhibit the gravitational effects of spin.
D. W. Sciama (Ithaca, N.Y.)

4523:
Olbert, Stanislaw. Motion of cosmic-ray particles in galactic magnetic fields. Proc. Sympos. Appl. Math., Vol. 9, pp. 10–18. American Mathematical Society, Providence, R.I., 1959.

After providing a brief outline about the known facts on the primary cosmic radiation, the author gives a short summary of Fermi's theory for the acceleration of cosmic radiation in outer space. *H. Messel* (Sydney)

4524:
Gusev, V. D. Some questions on the dispersion of radiowaves in the ionosphere. Radiotehn. i Elektron. **4** (1959), 12–16 (Russian); translated as Radio Engrg. and Electronics **4**, no. 1, 15–22.

Author's summary: "The part played by non-uniform waves in the angle spectrum of a dispersed field is ascertained when the ionosphere is illuminated by plane and divergent waves. The full time-space correlation function is given."

4525:
Gusev, V. D. The statistical properties of large ionospheric inhomogeneities. Radiotehn. i Elektron **5** (1960), 179–187 (Russian); translated as Radio Engrg. and Electronics **5**, no. 2, 1–13.

Author's summary: "The statistical and correlational properties of the phases of waves reflected from an ionosphere containing large inhomogeneities are considered. The analysis is carried out on the basis of solution of the eikonal equations. It is demonstrated that only with a normal distribution of angles of incidence of the wave is the distribution of rate of change of phase with respect to time independent of the distance to the ionosphere. The statistical character of focusing phenomena by large ionospheric inhomogeneities, which may have substantial significance in the phase method for studying these inhomogeneities, is investigated."

4526:
Budden, K. G.; Clemmow, P. C. Coupled forms of the differential equations governing radio propagation in the ionosphere. II. Proc. Cambridge Philos. Soc. **53** (1957), 669–682.

GEOPHYSICS

See also 4273, 4274, 4280, 4292, 4293, 4306, 4515.

4527:
Evison, F. F. On the growth of continents by plastic flow under gravity. Geophys. J. **3** (1960), 155–190.

4528:
Bullard, E. C. The automatic reduction of geophysical data. Geophys. J **3** (1960), 237–243.

4529:
Popova, A. G. Influence of the velocity of the wind on the distribution of the elements of waves. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 3, 83–92. (Russian)

4530:
Long, Robert R. A laminar planetary jet. J. Fluid Mech. **7** (1960), 632–638. (1 plate)

A theory is developed for jet motions in a rotating viscous fluid confined to a spherical layer. The jet is maintained by the balance between the inwards convection of vorticity from the rotating fluid and the diffusion by molecular or eddy viscosity. A linearised model is solved explicitly, and some qualitative deductions are made for cases where non-linear effects may be important. The reviewer believes this to be an important contribution in the understanding of the localised high velocity currents or jets prevalent in the atmosphere and the oceans. *O. M. Phillips* (Baltimore, Md.)

4531:
Gleeson, Thomas A. A prediction and decision method for applied meteorology and climatology, based partly on the theory of games. J. Meteorol. **17** (1960), 116–121.

Author's summary: "With the aid of examples, a prediction and decision method is described for use in applied meteorology or climatology where economic consequences have been evaluated beforehand. The predictand, which may be a single variable or several variables in combination, is divided into a finite number of forecast classes.

Confidence limits of the relative frequency of each predictand class are necessary. The mathematical theory of games of strategy is utilized, but elementary algebra suffices for all required computations.

"The method gives decisions and predictand frequency distributions that tend to minimize expenses (or maximize minimum gains) in ventures affected by weather or climate regardless of how uncertain either of the latter may be."

W. J. Pierson, Jr. (New York)

4532:

Bolotin, V. V. Statistical theory of aseismic structures. Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 4, 123–129. (Russian)

4533:

Nanda, J. N. Evaluation of average crustal characteristics from reverberation of seismic waves. Proc. Roy. Soc. London. Ser. A **256** (1960), 28–38.

When ground motion is measured at points close to the source of an explosion, reverberations are observed which are due to the back-scattering of P-waves from irregularities in the Earth's crust and from the surface. It is assumed in this paper that the velocities of the bodily elastic waves increase linearly with depth in the crust, the P-wave velocity at depth z below the base of the superficial layer of weathered material being $V_z = V_0 + az$, where a is a constant. Theoretical expressions are then derived for the variation with time T of the amplitude A_s of the vertical ground motion resulting from the following reverberation mechanisms: (i) surface scattering of surface waves; (ii) bodily scattering of P-waves without conversion; (iii) surface scattering of P-waves without conversion; (iv) surface scattering of P-waves with conversion into Rayleigh waves; (v) surface scattering of P-waves with conversion into S-waves. In case (i) it is found that $A_s \propto T^{-1/2}$, and in the other cases $A_s \propto \exp(-kaT)$ for $aT \gg 1$, where k takes the respective values 0.2, 0.5, 0.2, 0.6. Since reverberation characteristics are observed to vary from place to place mechanism (i) cannot predominate. When the other theoretical expressions are compared with measurements made near a shot point in Virginia, U.S.A., and the value $a = 0.075$ km/sec per km depth is adopted, the results are found to be in good agreement with the curve corresponding to mechanism (v), P to S surface scattering. The appropriateness of the assumed value of a is then verified by an examination of travel-time curves for this area.

Assuming, on the basis of these results, that the average velocity gradient for P-waves in the Eastern U.S.A. is 0.075 km/sec per km depth, the depth of penetration of the rays which give rise to the high intensity arrivals usually associated with critical reflections from the Mohorovičić discontinuity is computed, and found to be no more than 14 km. The author accordingly attributes the intense arrivals in this area not to reflection from a world-wide discontinuity but to a focusing effect caused by a peculiar change of velocity gradient at a depth of about 14 km. This interpretation implies the abandonment of the constant velocity layer as the basic element in the large-scale structure of the Earth's crust, and, if verified, will require some revisions in current theories of the crust.

P. Chadwick (Sheffield)

4534:

Bullen, K. E. A new method of deriving seismic velocity distributions from travel-time data. Geophys. J. **3** (1960), 258–269.

Author's summary: "A law of the form $v = ar^{\alpha}$ gives a useful first approximation to the seismic velocity distribution in various parts of the Earth. A method is here devised of finding the corrections to numerical values given by the law which are needed to fit a given set of seismic travel-time data. The method provides an alternative to the classical Herglotz method of determining seismic velocity distributions, and can be specially powerful in regions where the proportionate deviations from the above velocity law are small or moderate. The method is exact in spite of the non-linear form of the first approximation. Auxiliary notes are given which facilitate computation in particular cases, for example, when the form $T = a\Delta - b\Delta^3$ has been used to represent part of the travel-time data. For purposes of illustration, the method is applied to determining the P velocity distribution for the whole region E of the Earth. The method incidentally illustrates anew the importance of the parameters $\zeta = d \log v / d \log r$ and $\alpha = 2/(1-\zeta)$ in ray theory."

4535:

Zvyagin, B. M. Rock crushing followed by gravitational concentration. General theory and methods of computation. Zap. Leningrad. Gorn. Inst. **36** (1958), no. 3, 43–64. (Russian)

4536:

Žuravlev, P. A.; Zaharevič, A. F. Distribution of strains in a rock mass penetrated by a horizontal working with circular cross-section. Zap. Leningrad. Gorn. Inst. **36** (1958), no. 3, 101–105. (Russian)

4537:

Parker, E. N. Interaction of the solar wind with the geomagnetic field. Phys. Fluids **1** (1958), 171–187.

Author's summary: "The dynamical properties of the solar wind blowing past the geomagnetic field are investigated by considering the effective viscosity and the resulting transition layer thickness. The collision of ions in the solar wind produces a negligible viscosity in the flow past the geomagnetic field, but such an inviscid flow is shown to be unstable. The resulting disordered interface between the field and the wind yields Fermi acceleration of ions and consequently a not insignificant effective viscosity. The Fermi acceleration results in suprathermal ions which may have an energy spectrum like that observed for primary auroral protons.

"The auroral zones and the agitated nature of the polar geomagnetic field are shown to follow from the depth of penetration of the solar wind into the geomagnetic field. The injection of gas into the geomagnetic field is studied. The effect at Earth of the distortion of the outer boundary of the geomagnetic field is computed; no matter how unevenly and anisotropically the outer field is distorted, the effect at Earth is a nearly uniform perturbation field which is closely parallel to the geomagnetic axis. Pushing in on the outer field increases the horizontal component at Earth, and pulling out decreases it; the total increase

of the horizontal component is the algebraic sum of all the pushing and pulling. The simultaneous world-wide onset and the main phase of a geomagnetic storm follow.

"The common tendency of large and/or violent bodies of plasma to produce suprathermal particles is noted and suggested to be a general dynamical property."

W. J. Pierson, Jr. (New York)

4538:

Svešnikov, A. G.; Hapaev, M. M. On a problem of aero-electric surveying. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 2, 113–120. (Russian)

4539:

Tárczy-Hornoch, A. Über die mit Entfernungsmessung kombinierten Punkteinschaltungen. Acta Tech. Acad. Sci. Hungar. **30** (1960), 111–131. (English, French and Russian summaries)

A discussion of the elementary problem of performing point intersection and resection by using one or more measured sides as input instead of, or in addition to, observed angles or directions.

B. Chovitz (Washington, D.C.)

4540:

Linkwitz, Klaus. Beiträge zur Methodik der trigonometrischen Höhenmessung. Bayer Akad. Wiss. Math.-Nat. Kl. S.-B. **1959**, 263–288 (1960).

OPERATIONS RESEARCH, ECONOMETRICS, GAMES

See also 4111, 4208, 4531, 4588.

4541:

Marty, Alvin. The aggregate supply function. Metroecon. **11** (1959), 179–187.

4542:

Sengupta, S. Sankar. Behavior equations and force equations. Metroecon. **11** (1959), 168–178.

The author provides an economic interpretation of the partial differential equations of classical and relativistic physical dynamics. The applications relate to the static theory of production and economic innovations.

G. Tintner (Ames, Iowa)

4543:

König, Heinz; Ulrich, Herbert. Grundlagen und Bedeutung der Input-Output-Analyse. Schweiz. Z. Volkswirtschaft. Statist. **93** (1957), 77–91.

This article gives a critical survey of the development of input-output analysis to date. Some effort is made to evaluate the relation between the more general models of economic theory and the approximations applied in input-output studies. *T. Haavelmo* (Oslo)

4544:

Gerstenhaber, M. A procedure for finding a zero of a vector-valued function with certain monotonicity properties. J. Soc. Indust. Appl. Math. **8** (1960), 514–517.

The continuous vector function $v(x)=(v_1(x), \cdots, v_n(x))$ of a vector variable $x=(x_1, \cdots, x_n)$ is said to be of Leontief type with threshold R if (i) $\sum v_i(x)=0$, (ii) v_i is an increasing function of x_i and a decreasing function of x_j $(j\neq i)$, (iii) there is a real number R such that $x_j-x_i\geq R$ implies $v_i<0$. The author gives a convergent iterative procedure for solving $v(x)=0$, a problem proposed by Kelley. He also exhibits an infinitely differentiable function of Leontief type. *A. J. Hoffman* (New York)

4545:

Soroka, Walter W. Simulation in science and technology. Appl. Mech. Rev. **13** (1960), 621–624.

4546:

Koopman, B. O. Fallacies in operations research. Operations Res. **4** (1956), 422–426, comments by Charles Hitch, 426–430.

4547:

Morse, Philip M. Operations research. Frontiers of numerical mathematics, pp. 69–82. University of Wisconsin Press, Madison, Wis., 1960.

In contrast with the author's former definitions of operations research, which leaned toward the purely mathematical aspects of the science, the present formulation stresses operations research as an experimental science.

References and a few illustrations are made in this paper to various techniques employed in operations research, such as Markoff processes and simulation, and their applications to some practical situations, such as inventory processes. *A. Bakst* (New York)

4548:

Cobb, Howard. Operations research—a tool in oil exploration. Geophysics **25** (1960), 1009–1022.

Author's summary: "Operations research is a tool which may be used by the exploration man to help in planning exploration activities and in making decisions. Electronic computers and mechanized data handling may be used in the analyses which are a part of operations research. Various types of exploration data which we put on punch cards are readily available for operations research studies. Mathematical models of various activities play a large role. They are often built up from data of past experience. Thus, statistics and probability theory are important to the studies. Other techniques commonly associated with operations research studies are linear programming, game theory, queueing theory, Monte Carlo, and search theory. Operations research has been defined as quantitative common sense. It is probably the cheapest and one of the more effective of our exploration tools. We use it when we plan optimum control for a geophysical survey. We use it when we analyze the future of a frontier oil province."

4549:

Rinehart, R. F. Effects and causes of discrepancies in supply operations. Operations Res. **8** (1960), 543–564.

4550:

Dreyfus, Stuart E. A generalized equipment replacement study. J. Soc. Indust. Appl. Math. **8** (1960), 425–435.

Using the functional equation technique of dynamic programming, the author discusses various classes of equipment replacement problems in which technological improvement is taken into account. The computational feasibility of this approach is illustrated by means of some numerical solutions. References are given to related work by Bellman, Howard, Dreyfus, Sasieni and Terbrough.

R. E. Bellman (Santa Monica, Calif.)

4551:

Potter, Norman S. The optimization of astronautical vehicle detection systems through the application of search theory. Proc. IRE **48** (1960), 541–553.

From the author's summary: "A generalized theory of detection system performance is developed and applied to the analysis of collision warning and the optimal allocation of search effort for astronautical vehicles. The kinematic basis of the relative frequency of intercepts with randomly moving particles is presented. The rational choice of the frame time is investigated in terms of the information rate out of the sensor and the requirements of the decision-making apparatus. The very important case of contacts between astronauts and other objects independently moving on trajectories defined by a generalized central force field is separately considered."

J. Kiefer (Ithaca, N.Y.)

4552:

Bellman, Richard. Combinatorial processes and dynamic programming. Proc. Sympos. Appl. Math., Vol. 10, pp. 217–249. American Mathematical Society, Providence, R.I., 1960.

A survey of the use of functional equation techniques in discrete problems of transportation, scheduling, catering, sequential search, etc. *J. Kiefer* (Ithaca, N.Y.)

4553:

Romanovskiĭ, I. V. On a theorem of R. Bellman. Teor. Veroyatnost. i Primenen. **4** (1959), 456–458. (Russian. English summary)

The equations $\lambda y_i = \max_q \sum_{j=1}^{N} a_{ij}(q)y_j$ $(i = 1, 2, \cdots, N)$, and $x_i(n+1) = \max_q \sum_{j=1}^{N} a_{ij}(q)x_j(n)$, $x_i(0) = c_i$, $c_i \geq 0$, were treated by the reviewer in his book *Dynamic programming* [Princeton Univ. Press, Princeton, N.J., 1957; MR **19**, 820] and elsewhere. The author points out that there were lacunae in these discussions and supplies complete proofs.

R. E. Bellman (Santa Monica, Calif.)

4554:

Lambert, F. The traveling-salesman problem. Cahiers Centre Études Rech. Oper. **2**, 180–191 (1960).

In treating the traveling salesman problem, the author modifies the solution of the corresponding assignment problem by a series of steps that aim at eliminating loops while respecting the integral character of the solution. A small illustrative problem involving five cities is presented, but no comments are offered on the practicability of the proposed procedure for a problem of larger size.

W. Prager (Providence, R.I.)

4555:

Giffler, B.; Thompson, G. L. Algorithms for solving production-scheduling problems. Operations Res. **8** (1960), 487–503.

The paper discusses the scheduling of the production of m commodities on n facilities. A schedule is called feasible if it respects all precedence relations and optimal if its execution does not require more time than that of any other feasible schedule. It is shown that the optimal schedules form a subset of the set of "active" schedules (roughly speaking, the schedules with compact Gantt chart). An algorithm for generating all active schedules is given. Since their number would be very large in practical scheduling problems, it is suggested that a smaller number of random active schedules be constructed. It is stated that for a problem with $mn = 2000$, it took 20 minutes to compute 200 random active schedules on an IBM 704; while none of these needs to have optimum execution time, practical experience with the algorithm seems to have been good. Several modifications of the algorithm for problems with added restrictions are discussed.

W. Prager (Providence, R.I.)

4556:

Beale, E. M. L. On quadratic programming. Naval Res. Logist. Quart. **6** (1959), 227–243.

The author's algorithm [J. Roy. Statist. Soc. Ser. B **17** (1955), 173–184, 194–203; MR **19**, 619; pp. 173–177] for the computational solution of the quadratic programming problem of minimizing a convex function of degree two in finitely many nonnegative variables is described in detail and illustrated by the working out of a numerical example.

The simplex method for linear programming is described in terms sufficiently general as to clarify the motivation of the present algorithm: at any stage of the procedure, a non-basic variable with respect to which the partial derivative of the minimand is negative is introduced into the solution up to the point where (i) no more can be introduced without making some variable negative (the only possibility in the linear case), or (ii) the derivative vanishes. In each latter instance an auxiliary variable must be introduced in order to maintain the derivative at zero for some number of iterations; it may later disappear.

Comparison is made with another procedure [P. Wolfe, Econometrica **27** (1959), 382–398; MR **21** #5514] in general use; differing advantages are found for both.

P. Wolfe (Santa Monica, Calif.)

4557:

Houthakker, H. S. The capacity method of quadratic programming. Econometrica **28** (1960), 62–87.

A method is presented for assigning nonnegative values to several variables so as to maximize a given quadratic objective function while satisfying given linear constraints with nonnegative coefficients. The method is parametric. One takes the constraint $\sum x_i \leq \beta$, and gradually increases β from 0, keeping the optimum solution for the current value of β at all stages. The solution then traces out a finite set of straight line segments. The method has similarities with that of the reviewer [J. Roy. Statist. Soc. Ser. B **17** (1955), 173–184; MR **19**, 619; see also preceding review]. But the author does not introduce

additional (free) variables to retain the Simplex form. {A similar parametric problem is considered on pp. 238–241 of the paper reviewed above.}

E. M. L. Beale (Teddington)

4558:

Pütter, Paul Stefan. Ein allgemeines Maximalisierungverfahren. Z. Angew. Math. Mech. **39** (1959), 466–472. (English, French and Russian summaries)

The author describes a "gradient" procedure which it is claimed will solve the problem: $f(\mathbf{x})=\max$; $g_\mu(\mathbf{x})=0$, $\mu=1, 2, \cdots, M<N$, $\mathbf{x}\in R_N$, where R_N is some rectangular domain in E^N. Assuming a point $\mathbf{x}_0$ on the constraint hypersurfaces to be known (with no discussion of its determination), an iteration is proposed to find an $\mathbf{x}_1$ such that $f(\mathbf{x}_1)>f(\mathbf{x}_0)$ and $g_\mu(\mathbf{x}_1)=0$. For this purpose the projection of $\nabla f(\mathbf{x}_0)$ on the tangent hyperplanes to the $g_\mu(\mathbf{x})$ at $\mathbf{x}_0$ is employed. However, no convergence proofs are given and indeed the lack of any conditions such as concavity or even regularity of $f(\mathbf{x})$ cast doubt on the general applicability of the method. Previous results on gradient methods and even nonlinear programming are not referred to in the paper. It is stated that the method has been used satisfactorily, but no results are presented.

H. B. Keller (New York)

4559:

Bertrais, Jean. Extremum d'une somme ou d'un produit de fonctions. C. R. Acad. Sci. Paris **250** (1960), 2115–2117.

The problem of maximizing the function $\sum_{i=1}^{N} f(x_i)$ over all x_i satisfying the constraints $x_i\geqq 0$, $\sum_{i=1}^{N} x_i=a$ leads by way of the techniques of dynamic programming to the study of the recurrence relation $F_N(a)=\max[f(x)+F_{N-1}(a-x)]$, where the maximum is over $0\leqq x\leqq a$. Under the assumption that a unique internal maximum exists which can be found by use of calculus, one obtains the result that $F_N(a)=Nf(a/N)$.

R. E. Bellman (Santa Monica, Calif.)

4560a:

Berkovitz, L. D.; Dresher, Melvin. A game-theory analysis of tactical air war. Operations Res. **7** (1959), 599–620.

4560b:

Berkovitz, Leonard D.; Dresher, Melvin. A multimove infinite game with linear payoff. Pacific J. Math. **10** (1960), 743–765.

Both papers present the same results, the second with more mathematical emphasis. The authors analyze a quite general multimove zero-sum two-person game with a linear payoff function, and succeed in deriving an explicit solution for the optimal strategies for both players and for the value of the game. The game is symmetric, except that the initial conditions for the two players are different; one player has an optimal pure strategy but the other player must randomize on the strategies. The paper includes a table showing the numerical solutions for games with eight or less moves. The game had its origin in a military problem concerning allocation of resources among several tasks, and this application is discussed by the authors in the first paper.

M. M. Flood (Ann Arbor, Mich.)

4561:

Aumann, Robert J. Acceptable points in games of perfect information. Pacific J. Math. **10** (1960), 381–417.

This is the second of a series of papers on acceptable points; for the first see Aumann, *Contributions to the theory of games, Vol. IV*, pp. 287–324 [Princeton Univ. Press, Princeton, N.J., 1959; MR **21** #3274]. The main result is that the acceptable points (if any) of a game with perfect information are precisely the supergame payoffs to strong equilibrium points in pure (but not stationary) strategies. The author conjectures that the game structures for which this is true independently of the payoff are just the essentially determinate structures whose complete inflations have effectively perfect information. Finally, he formulates a "non-cooperative" notion of acceptability in which correlated strategies are excluded (throughout this paper and the preceding one, side payments are excluded), and shows that for games of perfect information this coincides with the "cooperative" notion. With a corresponding restriction on supergame strategies, the main theorems of this paper and the preceding one remain valid in the non-cooperative formulation for the case of perfect information.

J. Isbell (Seattle, Wash.)

BIOLOGY AND SOCIOLOGY

See also A3696, 4135, 4137.

4562:

Macey, Robert I.; Wolf, A. V. Kinetics of ultrafiltration hemodialysis. Bull. Math. Biophys. **22** (1960), 217–226.

4563:

Scheer, Bradley T. The flux-force relations across complex membranes with active transport. Bull. Math. Biophys. **22** (1960), 269–284.

Author's summary: "Equations are derived for the total material flux, and the total electric current flux, across a complex membrane system with active transport. The equations describe the fluxes as linear functions of forces across the system, and specifically of electrical potential, hydrostatic pressure, chemical potentials, and active transport rates. The equations can be simplified for experimental studies by making one or more of the forces equal to zero. The osmotic pressure difference across a membrane system is shown to be a function of the electrical potential and chemical potential differences and of the active transport rates. The transmembrane potential is shown to be the sum of a diffusion potential and an active transport potential. A simple equation is derived describing the current across a membrane as a linear function of the electrical potential and the active transport rate. Specific examples of the application of the equations to nerve membrane potentials are considered."

4564:

Andreoli, Giulio. Algebre non associative e sistemi differenziali di Riccati in un problema di genetica. Ann. Mat. Pura Appl. (4) **49** (1960), 97–116.

Genetics of populations is studied by means of nonassociative algebra, the basic idea being the same as that

of Glivenko [Dokl. Akad. Nauk SSSR **13** (1936), 371–372] and the reviewer [Proc. Roy. Soc. Edinburgh **59** (1939), 242–258; ibid. Sect. B **61** (1941), 24–42; MR **1**, 99; **2**, 237]. A population as a distribution of genotypes $A^{(r)}$ ($r=1, \cdots, N$) in proportions $\alpha^{(r)}$ ($\sum \alpha^{(r)}=1$) is represented by a hypercomplex number $U=\sum \alpha^{(r)}A^{(r)}$. The symbols $A^{(r)}$ are subject to a commutative nonassociative multiplication table $A^{(r)}\times A^{(s)}=\sum c_{rst}A^{(t)}$ showing the probability distribution of genotypes in the offspring of each type of mating. Under the simplest assumptions including random mating and distinct generations, passage from the nth to the $(n+1)$th generation of U is given by (1) $U_{n+1}=U_n^2$; and the author considers also the cases (2) $U_{n+1}=\rho U_n^2+(1-\rho)U_n$ $(0<\rho<1)$ (partial overlapping of generations), (3) $U_{n+1}=\rho U_n^2+\sigma U_n+\tau V_n$ $(\rho+\sigma+\tau=1)$ (the same with influx from another population V), and the analogous cases (1′), (2′), (3′) which arise when there is no gap between generations and U varies continuously in time. In case (2′), for example, the hypercomplex number U (with surrender of the condition $\sum \alpha^{(r)}=1$) obeys a differential equation of Riccati type $U'(t)=kU^2(t)+lU(t)$.

Consequences are analyzed in detail for the special case of simple mendelian inheritance, where the number of genotypes is $N=3$ (dominant, heterozygote, recessive); and the author also examines, for case (1) of this, a generalization of the nonassociative algebra which he does not interpret biologically but which would represent the effects of selection. {The author does not appreciate the significance of his conclusions in the various cases. In so far as they can be interpreted biologically they are almost all different ways of expressing that gene frequencies are either constant, or not constant, as the case may be, and may or may not be the same for the populations U and V. Another way of expressing this criticism is to say that, in the reviewer's terminology, it is simpler where possible to work in the gametic rather than the zygotic algebra. It should be added that in one section of the paper (pp. 112–113) the author seems to be discussing inheritance in autopolyploids, but the equations obtained are not correct for this, being in fact formally those which apply in the case of multiple alleles.}

I. M. H. Etherington (Edinburgh)

4565:

Simon, Herbert A. ★Models of man, social and rational. Mathematical essays on rational human behavior in a social setting. John Wiley & Sons, Inc., New York; Chapman & Hall, Ltd., London, 1957. xiv+287 pp. $5.00.

4566:

Hoffmann, Hans. Symbolic logic and the analysis of social organization. Behavioral Sci. **4** (1959), 288–298.

By means of symbolic logic notation, an algebra of kinship relations in the Pawnee tribe is established. Several involved relations are formally defined in terms of a few fundamental ones. For example, "'tiwatsiriks' are males who are same generation agnatic kinsmen of any of ego's ascending generation uterine kinsmen". The notation, the explicit institutionalized rules (of nominal spouse-ship cohabitation, marriage, residence, etc.), and the rules of deduction permit the establishment of implicit rules as theorems. In particular, the author formally proves the following theorem, which, although previously conjectured, is not found as an explicit institutionalized rule in the ethnographic record: "A Pawnee's marriage partner must be the granddaughter of the sister of ego's father's father."

A. Rapoport (Ann Arbor, Mich.)

4567:

Rashevsky, N. On imitative behavior. Bull. Math. Biophys. **22** (1960), 63–71.

Various models of mass imitative behavior previously proposed by the author are compared. In all of them the population is partitioned into classes X and Y, according to whether their members perform one of two mutually exclusive acts at a given time. Depending on the influence of others, individuals change their behavior from one act to another. In some models, certain "active" individuals X_0 and Y_0 are present who are immune to such influences.

In the simplest linear model, the time derivatives of X and Y are linear combinations of X, Y, X_0, and Y_0. In another model the coefficients of X_0 and Y_0 are linearly decreasing with X and Y respectively (denoting diminishing efforts of the actives to convert the passives). This leads to the same type of equations (with different coefficients). The author's statement, following equation (8), that X/Y varies continuously with X_0/Y_0 is not correct if one interprets it as saying $(X/Y)_{\text{equilibrium}}=f(X_0/Y_0)$, since (X/Y) at equilibrium is a quotient of two polynomials of degree one in X_0 and Y_0 which are not homogeneous. Moreover the reader should be reminded that the equilibrium point of equation (8) is stable only if $2a_1<a_0^*\varepsilon X_0+C_0^*\varepsilon' Y_0$. The "threshold", therefore, is not a value of X_0/Y_0 as the author states, but a value of $a_0^*\varepsilon X_0+C_0^*\varepsilon' Y_0$. In the simplest linear case, where $\varepsilon=\varepsilon'=0$, the equilibrium is always unstable: the passives are converted either to one form of behavior or to another. Equation (4) contains a misprint and should read $X+Y=N'$. The above-mentioned inaccuracies have no bearing on the main results of the paper, which follow.

In still another model, the time derivative of a certain propensity to perform an act is linear in $(X-Y)$ with a decay term. The probability of performing an act is computed from this propensity and from the initial inherent propensity distributed through the population. If the inherent distribution is symmetric with respect to zero, the time derivative is expandable in odd powers of $(X-Y)$. Thus the first linear model is shown to be a first approximation to the present one.

Taking the cubic term into account, a graphical phase-space solution of the non-linear model is given and the stability properties of its equilibria are discussed.

A. Rapoport (Ann Arbor, Mich.)

4568:

Lienert, G. A.; Ebel, O. Ein Index zur numerischen Bestimmung der Niveau-Eigenschaft eines psychologischen Tests. Metrika **3** (1960), 117–127. (English summary)

4569:

Ashby, W. Ross. ★Design for a brain: The origin of adaptive behaviour. 2nd ed., revised. John Wiley & Sons, Inc., New York; Chapman & Hall, Ltd., London; 1960. ix+286 pp. $6.50.

In the eight years since the appearance of the first

edition [Wiley, New York, 1952] a considerable amount of data and understanding of the mechanism concerning brain-processes has been acquired. In addition, information theory has become a well-developed discipline, and extensive research dealing with the nature of self-organizing systems, e.g., the human brain, has been launched [M. Yovits and S. Cameron, Editors, *Self-organizing systems*, Pergamon Press, New York, 1960; MR **22** #4570].

The over-all purpose of this mathematico-biological treatise has not been altered from that of the first edition, although only the first third of the book has not been rewritten. Thus, the book still claims to pose a solution to a specific problem, viz., "the origin of the nervous system's unique ability to produce adaptive behaviour".

The treatment of system dynamics, stability and ultra-stability is essentially the same as in the first edition, i.e., based upon a study of the differential equations of mathematical physics. The new chapters include "the homeostat", "the recurrent situation", e.g., accumulation of adaptations, "the system with local stabilities", "adaptation in the multistable system" and "amplifying adaptation".

The appendix to the book contains all of the technical mathematical details. Here the author follows the definitions, but not the terminology, of N. Bourbaki [e.g., *Fonctions d'une variable réelle*, Chap. IV–VII, Hermann, Paris, 1951; MR **13**, 631].

{By way of criticism to the organization of this fine book, the reviewer points out that (i) although the second edition is an enlargement of the first edition the index of the second edition has only about one-half as many entries as the first edition, (ii) the section of Chapter 18 entitled "the origin of adaptive behaviour" which is listed in the table of contents, does not appear in the text, (iii) under Chapter 22 in the table of contents one should add "systems with multiple fields" which does appear in the text and (iv) more than once the heading of a subsection of a chapter is the same as that of the chapter itself.}

A. A. Mullin (Urbana, Ill.)

4570:

Yovits, Marshall C.; Cameron, Scott. (Editors) ★Self-organizing systems. Proceedings of an interdisciplinary conference, 5 and 6 May, 1959. International Tracts in Computer Science and Technology and Their Application, Vol. 2. Pergamon Press, Oxford-London-New York-Paris, 1960. ix + 322 pp. $8.50.

This book represents the official proceedings of an interdisciplinary conference on the subject of the fast-growing field of cognitional systems that was held in Chicago, Ill., on 5 May and 6 May of 1959. The text of this well-edited work consists of 13 technical papers contributed by 15 eminent researchers from the disciplines of philosophy, electrical engineering, physiology and psychology.

The specific titles and authors of papers in their order of appearance in the book (this order is roughly inverse to the order of generality of the paper) are the following: (1) "Self-organizing models for learned perception" by B. G. Farley, (2) "On self-organizing systems and their environments" by H. M. von Foerster, (3) "Statistical models for recall and recognition of stimulus patterns by human observers" by W. K. Estes, (4) "Perceptual generalization over transformation groups" by F. Rosenblatt, (5) "The organization and reorganization of embryonic cells" by R. Auerbach, (6) "Further consideration of cybernetic aspects of homeostasis" by S. Goldman, (7) "Feedback through the environment as an analog of brain functioning" by G. H. Bishop, (8) "A variety of intelligent learning in a general problem solver" by A. Newell, J. C. Shaw and H. A. Simon, (9) "Learning in neural systems" by P. M. Milner, (10) "Blind variation and selective survival as a general strategy in knowledge-processes" by D. T. Campbell, (11) "The natural history of networks" by G. Pask, (12) "The reliability of biological systems" by W. S. McCulloch, and (13) "Computation, behavior, and structure in fixed and growing automata" by A. W. Burks. In addition there is the text of an after-dinner speech by A. M. Uttley entitled "The mechanization of thought processes".

Papers (1), (2), (3), and (4) consider "perception of the environment". Papers (5), (6), and (7) treat "effects of environmental feedback". Papers (8), (9), and (10) deal with "learning in finite automata". Papers (11), (12), and (13) are concerned with "structures of self-organizing systems".

{The reviewer recommends the book to all cybernetically-minded engineering scientists.}

A. A. Mullin (Urbana, Ill.)

4571:

Roy, Archie E. On a method of storing information. Bull. Math. Biophys. **22** (1960), 139–168.

This paper, which adopts an "activity" theory of memory [(i) N. Rashevsky, *Mathematical biophysics*, Univ. of Chicago Press, Chicago, Ill., 1938; (ii) W. S. McCulloch and W. Pitts, same Bull. **5** (1943), 115–133; MR **6**, 12], is concerned with some of the properties of an elementary mathematical model for the storage of information within ideal neural nets as specified by four postulates.

The theory is a statistical one [see e.g., A. Shimbel and A. Rapoport, ibid. **10** (1948), 41–55; MR **9**, 521], and considers memory a global, rather than a local, property.

A. A. Mullin (Urbana, Ill.)

INFORMATION AND COMMUNICATION THEORY

See also 4528.

4572:

Chintschin, A. J.; Faddejew, D. K.; Kolmogoroff, A. N.; Rényi, A.; Balatoni, J. ★Arbeiten zur Informationstheorie, I. Mathematische Forschungsberichte, IV. VEB Deutsche Verlag der Wissenschaften, Berlin, 1957. 134 pp. DM 18.40.

German translations of the following articles: Hinčin, Uspehi Mat. Nauk **8** (1953), no. 3 (55), 3–20 [see #4573].

Hinčin, ibid. **11** (1956), no. 1 (67), 17–75 [MR **17**, 1098].

Faddeev, ibid. 227–231 [MR **17**, 1098].

Kolmogorov, Sess. Akad. Nauk SSSR. Probl. Avtomat. Prolzvod., 15–20 Oct. 1956, pp. 66–99; Chap. I, II [cf. Trans. IRE **IT2** (1956), 102–109; also MR **21** #6297].

Rényi and Balatoni, Magyar Tud. Akad. Mat. Kutató Int. Közl. **1** (1956), 1–2, 9–40.

4573:

Hinčin, A. Ya. The concept of entropy in the theory of probability. Amer. Math. Soc. Transl. (2) **12** (1959), 181–197.

The Russian original [Uspehi Mat. Nauk (N.S.) **8** (1953), no. 3 (55), 3–20] was reviewed in MR **15**, 238.

4574a:

Gel'fand, I. M.; Yaglom, A. M. Calculation of the amount of information about a random function contained in another such function. Amer. Math. Soc. Transl. (2) **12** (1959), 199–246.

4574b:

Chiang, Tse-Pei. Remark on the definition of the quantity of information. Amer. Math. Soc. Transl. (2) **12** (1959), 247–250.

The original papers in Russian [Uspehi Mat. Nauk (N.S.) **12** (1957), no. 1 (73), 3–52; Teor. Veroyatnost. i Primenen. **3** (1958), 99–103] have already been reviewed [MR **18**, 980; **20** #789].

4575:

Bol'šakov, I. A. Passage of regular and random signals through a phase detector of commutation type. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1958**, no. 6, 45–70. (Russian)

4576:

Weiss, Lionel. On the strong converse of the coding theorem for symmetric channels without memory. Quart. Appl. Math. **18** (1960/61), 209–214.

Let Γ be a memoryless channel with binary input and output space the real line. Assume $f_0(y)$, $f_1(y)$ to be the probability density functions of the output Y corresponding to the transmission of 0, 1, respectively. Γ is called symmetric if the distribution of

$$2[f_0(Y)+f_1(Y)]^{-1}f_0(Y)$$

when Y has density function $f_0(y)$ is the same as that of $2[f_0(Y)+f_1(Y)]^{-1}f_1(Y)$ when Y has density function $f_1(y)$. The author shows, for a discrete binary symmetric channel, and for the semi-continuous symmetric channel defined above and satisfying an additional technical condition, the following: For any λ, $0 \leq \lambda < \frac{1}{2}$, there is a $K_\lambda > 0$ depending upon the channel, such that for all sufficiently large n the length L of any code of word length n and probability of error $\leq \lambda$ satisfies $\log_2 L \leq nC - K_\lambda n^{1/2}$, where C is the capacity of the channel. This result is stronger than previously published results, in which the existence of a negative K_λ (but for arbitrary discrete memoryless channels) was obtained. *A. Feinstein* (Urbana, Ill.)

4577:

Gilbert, E. N. Capacity of a burst-noise channel. Bell System Tech. J. **39** (1960), 1253–1265.

As a model of a channel with burst-noise, the author considers a particular binary symmetric channel with memory. The channel characteristics are determined by a Markov chain with two states G and B. In state G transmission is error free; in B error occurs with probability $1-h$. The capacity is explicitly determined in terms of and graphed for various values of the channel parameters. Formulas for the average recurrence time of errors as well as the covariance function of the noise process are given, and comparisons are made with recent measurements carried out on telephone circuitry.

A. Feinstein (Urbana, Ill.)

4578:

Sragovič, V. G. Optimum signal detection in the presence of correlated gaussian noise. Radiotehn. i Elektron. **4** (1959), 745–754 (Russian); translated as Radio Engrg. and Electronics **4**, no. 5, 16–28.

Author's summary: "A solution is given of the problem of finding the best method of detecting a gaussian signal of special form in the presence of correlated gaussian noise, and its probability characteristics are calculated."

4579:

Malahov, A. N. The spectrum of flicker noise. Radiotehn. i Elektron. **4** (1959), 54–62 (Russian); translated as Radio Engrg. and Electronics **4**, no. 1, 86–100.

Author's summary: "A short survey is given of the basic experimental and theoretical results concerning flicker noise in various systems and the fundamental difficulties in the theory of flicker noise are discussed.

"The possibility of overcoming these difficulties by assuming a non-stationary nature for flicker noise are examined. Considerations are put forward concerning the causes and localization of flicker noise."

4580:

Bašarinov, A. E.; Fleišman, B. S. The application of sequential analysis to binary communication systems with a Rayleigh distribution of signal intensity fluctuations. Radiotehn. i Elektron. **4** (1959), 155–160 (Russian); translated as Radio Engrg. and Electronics **4**, no. 2, 1–9.

Authors' summary: "Systems for processing fluctuating signals, using sequential analysis are considered. The average time characteristic and the probability of correct signal detection are calculated for the case of transmission with passive spaces, when the probability of false response (false alarms) is small in comparison with the permissible probability of missing the signal. The probability distribution of the number of completed tests is evaluated."

4581:

Kotel'nikov, V. A. Signals with maximum and minimum probability of detection. Radiotehn. i Elektron. **4** (1959), 354–358 (Russian); translated as Radio Engrg. and Electronics **4**, no. 3, 5–11.

Author's summary: "This article gives those signals which have maximum and minimum probability of detection by an ideal receiver. The evaluation of these probabilities is given."

4582:

Železnov, N. A. Some problems of the spectral-correlation theory of non-stationary signals. Radiotehn. i

Èlektron. 4 (1959), 359–373 (Russian); translated as Radio Engrg. and Electronics 4, no. 3, 12–30.

Author's summary: "For the description of non-stationary signals we introduce energetic characteristics connected with the present moment of time, and time-average characteristics, also frequency correlation functions characterizing the correlative relationship between the harmonic components of a random spectrum. Relationships are formulated between the corresponding spectral and correlation functions in the form of Fourier transformation pairs. The ergodic theorems are proved for non-stationary signals and the physical content of the conditions for which these theorems are valid is explained. The spectral and correlational functions of a few classes of non-stationary signals are examined."

4583:

Kuklev, L. P. Influence of noise fluctuations on a decoding unit during the intervals separating the pulses in a code group. Radiotehn. i Èlektron. 4 (1959), 374–380 (Russian); translated as Radio Engrg. and Electronics 4, no. 3, 31–40.

Author's summary: "On the basis of expressions for the average number of spurious responses of the decoding unit and on the basis of the signal transmission probability, the effect of various parameters of the receiver system on the noise-proof-feature of decoding is determined."

4584:

Doluhanov, M. P. On certain possible methods of measuring the entropy of discrete and continuous message sources. Radiotehn. i Èlektron. 4 (1959), 559–565 (Russian); translated as Radio Engrg. and Electronics 4, no. 4, 1–10.

Author's summary: "Three possible methods of automating the process of measuring the entropy of message sources are considered: (1) with the use of an electronic counter; (2) with the use of a decorrelator and subsequent application of a counter; and (3) measurement of the correlation function and calculation of the upper limit of entropy. The mathematical basis for the third method is presented."

4585:

D'yakov, Yu. E. Passage of signal and noise through a non-linear inertial system. Radiotehn. i Èlektron. 4 (1959), 936–941 (Russian); translated as Radio Engrg. and Electronics 4, no. 6, 38–46.

Author's summary: "A general method of linearizing equations describing fluctuations in non-linear narrow-band systems is considered for large signal/noise ratios."

4586:

Cybakov, B. S. The channel capacity of a two-path communication channel. Radiotehn. i Èlektron. 4 (1959), 1116–1123 (Russian); translated as Radio Engrg. and Electronics 4, no. 7, 66–77.

Author's summary: "The channel capacity of a two-path communications channel with gaussian additive noise is investigated. A simple expression has been obtained for the capacity in the case of uniform noise spectral density. It is shown that a reduction in capacity caused by two paths does not exceed 15 per cent."

4587:

Holmes, James F. Undetected errors in 5-unit code transmission and their elimination. Computers and Automation 9 (1960), no. 11, 10–13.

4588:

McCluskey, E. J., Jr. Error-correcting codes—a linear programming approach. Bell System Tech. J. 38 (1959), 1485–1512.

The author's main objective is to obtain the parity check matrices corresponding to systematic error-correcting codes of least redundancy having a specified minimum distance d (i.e., at least d disagreements) between any pair of code words. A characterization of these matrices is proved, and a linear program based on it is described for discovering such matrices. This linear program can be solved by a modification of the simplex method constrained to yield integer solutions. When the code parameters satisfy certain additional requirements, explicit solutions in closed form are obtained for these codes. With binary words of length n and with $2d > n$, code dictionaries attaining M. Plotkin's bound [*Binary codes with specified minimum distance*, Research Division Report 51–20, Moore School, Univ. of Pennsylvania, 1951] exist whenever $2d/(2d-n)$ is a power of 2. For systematic codes, this condition is also necessary. Examples of these codes can be obtained by simple modification of the Reed-Muller codes. It could be pointed out that if the constraint of systematicness were dropped, other codes attaining Plotkin's bound would exist, corresponding to all the Hadamard (orthogonal block-design) matrices.

S. W. Golomb (Pasadena, Calif.)

4589:

Culbertson, James T. Automation—its evolution and future direction. I. Computers and Automation 9 (1960), no. 11, 14–18.

4590:

Fatehchand, Richard. Machine recognition of spoken words. Advances in computers, Vol. 1, pp. 193–229. Academic Press, New York, 1960.

The use of computers for machine recognition of speech is shown to have been very restricted so far. In order to utilise a computer, the speech must be changed into a form suitable for input and this form must then be recognised. The mere matching of words with ones stored inside a computer would require a larger store than is practicable, and in order to introduce further methods, it is necessary to analyse speech processes logically.

Speech sounds may be divided into phonetic groups; however, the variation within these groups is sufficient to make recognition impossible. Further information may be supplied by an analysis of constraints on sounds. The storage space required for the statistical data on these constraints is again very large.

"Binary Classification Method" is proposed which involves deciding at each stage which of two opposite

characteristics a sound possesses: this method is well suited for use on a computer.

The paper ends with a description of three contemporary speech recognition machines, none of which is capable of dealing with continuous speech. The first machine recognises words in a small group, taking each word as a whole, and the second splits the word phonetically. The third machine can recognise members of a larger group of words, but a limited number of sounds; it divides sounds into phonetic groups on input and makes further identification with the aid of data on constraints.

This paper gives very clearly the background to the subject and contains many diagrams and graphs.

A. D. Booth (London)

SERVOMECHANISMS AND CONTROL

See also A3746, 4129, 4135, 4570.

4591:

Kuzovkov, N. T. A method for investigation of systems of automatic regulation containing linear links with variable parameters. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1958**, no. 6, 19–30. (Russian)

4592:

Zaborszky, John; Diesel, John W. A statistically averaged error criterion for feedback-system synthesis. J. Aero/Space Sci. **27** (1960), 128–134, 160.

The authors discuss some feedback control processes involving linear equations and quadratic error criteria.

R. E. Bellman (Santa Monica, Calif.)

4593:

Desoer, C. A. The bang bang servo problem treated by variational techniques. Information and Control **2** (1959), 333–348.

The variational problem is that of determining the vector $f(t)$ subject to constraints $|f_i(t)| \leq m_i$, $t \geq 0$, so as to transform the solution of $y' = Ay + f(t)$ from an initial state $y(0) = c_1$, to a terminal state $y = c_2$ in minimum time. A solution of particular cases of this problem was given by Bellman, Glicksberg and Gross [Quart. Appl. Math. **14** (1956), 11–18; MR **17**, 1206] and extended and generalized by a number of Russian authors and by LaSalle [Proc. Nat. Acad. Sci. U.S.A. **45** (1959), 573–577; MR **21** #4065]. The author shows that the problem can be treated by classical variational techniques, discusses a number of interesting applications and gives some numerical results. For an approach by dynamic programming techniques, see R. Bellman, *Adaptive control processes: A guided tour* [Princeton Univ. Press, Princeton, N.J., 1961].

R. E. Bellman (Santa Monica, Calif.)

4594:

Pavlov, A. A. Optimum transient processes in systems with a restricted third derivative. Avtomat. i Telemeh. **20** (1959), 1020–1036 (Russian. English summary); translated as Automat. Remote Control **20** (1960), 992–1007.

Author's summary: "Optimum transient processes in a system with a restricted third derivative of the controlled variable are examined. The form of optimum transient processes due to step disturbances is determined and a possible method of synthesizing the optimum controlling part of the system is given."

4595:

Kuhtenko, V. I. On designing correcting circuits for automatic control systems in accordance with the mean square error criterion. Avtomat. i Telemeh. **20** (1959), 1180–1187 (Russian. English summary); translated as Automat. Remote Control **20** (1960), 1151–1159.

Author's summary: "A method is presented for finding the optimal (in the sense of minimum mean square error) transfer function, the degree of whose numerator is a given number of units less than the degree of the denominator, and a discussion is given of the relationship of this question with the question of physical realizability of the correction circuit."

4596:

Komarnickaya, O. I. Application of the residue theory to transformation of automatic control systems equations to canonic variables. Prikl. Mat. Meh. **23** (1959), 845–850 (Russian); translated as J. Appl. Math. Mech. **23**, 1209–1217.

The author discusses the problem of obtaining explicit representations of the solutions of $dx/dt = Ax$ in terms of the characteristic roots and the characteristic polynomial $D(\lambda) = |A - \lambda I|$. *R. E. Bellman* (Santa Monica, Calif.)

4597:

Constantinescu, Paul. Systèmes de réglage automatique, dont le plan des phases présente plusieurs feuilles. Com. Acad. R. P. Romîne **9** (1959), 1129–1134. (Romanian. Russian and French summaries)

Author's summary: "L'auteur étudie des problèmes préliminaires à l'étude de la stabilité des systèmes de réglage dont le plan des phases présente plusieurs feuilles. Il cherche à établir le nombre minimum d'éléments que l'on doit connaître pour que les autres éléments, qui définissent le comportement du système, en résultent d'une manière unique."

4598:

Kalyaev, A. V. Computing the transient response in linear systems by the method of reducing the order of the differential equation. Avtomat. i Telemeh. **20** (1959), 1171–1179 (Russian. English summary); translated as Automat. Remote Control **20** (1960), 1141–1150.

Author's summary: "The paper considers the computation of the transient responses in linear systems with constant parameters by the method of reducing the order of the differential equation. Examples are given of the application of the method developed."

4599:

Kinyapin, S. D.; Neĭmark, Yu. I. On the stability of a relay system's equilibrium state. Avtomat. i Telemeh. **20** (1959), 1153–1162 (Russian. English summary); translated as Automat. Remote Control **20** (1960), 1121–1131.

4600:

Taft, V. A. On the analysis of the stability of the periodic modes of operation in nonlinear control systems with many degrees of freedom. Avtomat. i Telemeh. **20** (1959), 1163–1170 (Russian. English summary); translated as Automat. Remote Control **20** (1960), 1132–1140.

Author's summary: "On the basis of a generalized corollary of Hill's equation, a derivation is given of the characteristic equation (in finite form) of a system with many degrees of freedom whose parameters are periodic functions of time. The results obtained permit the use of the well-known Mikhailov criterion for analyzing the stability of the periodic modes of operation."

4601:

Mal'čikov, S. V. On the synthesis of linear automatic control systems with variable parameters. Avtomat. i Telemeh. **20** (1959), 1588–1594 (Russian. English summary); translated as Automat. Remote Control **20** (1960), 1543–1549.

4602:

Orurk, I. A. Synthesizing the elements of linear automatic control systems. Avtomat. i Telemeh. **20** (1959), 1595–1602 (Russian. English summary); translated as Automat. Remote Control **20** (1960), 1550–1557.

4603:

Faury, Bernard. Détermination de la fonction de transfert d'un servomécanisme. C. R. Acad. Sci. Paris **251** (1960), 196–197.

Author's summary: "Description d'une méthode pratique pour la détermination de la fonction de transfert d'un asservissement dont on connaît le schéma fonctionnel."

4604:

Bol'šakov, I. A. Analysis of tracking failure in automatic control systems in the presence of fluctuating noise. Avtomat. i Telemeh. **20** (1959), 1611–1622 (Russian. English summary); translated as Automat. Remote Control **20** (1960), 1566–1576.

Author's summary: "The failure of tracking in automatic control systems under the action of intense fluctuating noise is investigated by means of the Fokker-Planck equation. The Ritz-Galerkin method is used to solve the boundary problem where, by a failure, we understand an increase of the error in the system above some definite quantity. For a system with a smoothing circuit in the form of an integrator, formulas are found for the probabilities of absence of failures in the system as functions of the system parameters, noise level and time. The results are used in the analysis of the noise stability of the AFC (automatic frequency control) system of a receiver of continuous signals."

4605:

Solodovnikov, V. V.; Uakov, A. S. A frequency method for determining the dynamic characteristics of objects of automatic control from data on their normal usage. Avtomat. i Telemeh. **20** (1959), 1579–1587 (Russian. English summary); translated as Automat. Remote Control **20** (1960), 1533–1542.

Authors' summary: "A frequency method is presented for determining the dynamic characteristics of objects of automatic control from data obtained in the course of their normal usage, specifically, correlation and cross-correlation functions. The paper considers linear objects with many inputs and outputs and with constant lags, and also multiloop linear systems in the presence of noise. It is assumed for the analysis that the random processes occurring in the systems are ergodic and stationary, and that the objects under investigation are stable."

4606:

Batkov, A. M. Generalization of the shaping filter method to include nonstationary random processes. Avtomat. i Telemeh. **20** (1959), 1081–1094 (Russian. English summary); translated as Automat. Remote Control **20** (1960), 1049–1062.

Author's summary: "The paper considers the basic properties of the impulsive responses of linear systems with variable parameters of general form, and the correlation functions of the random processes at their outputs when acted upon by 'white noise'. The problem of determining the characteristics of the shaping filters for this class of non-stationary processes is solved, and a methodology is suggested for using them in simulation problems in analyzing the dynamic accuracy of systems."

4607:

Krasovskiĭ, N. N. On optimum control in the presence of random disturbances. Prikl. Mat. Meh. **24** (1960), 64–79 (Russian); translated as J. Appl. Math. Mech. **24**, 82–102.

The author considers the linear control problem described by the equation $dx/dt = Ax + By + r$ where $r = r(t)$ is a random vector, x is the state vector and y is the control. The general question considered is that of determining y subject to various constraints so as to maximize the probability of attaining certain goals. The question of the existence of optimal control is discussed, as well as certain necessary conditions. It is shown how Lyapunov functions may be used in this study, and a second order system is discussed using the geometric interpretation of Lyapunov functions due to Četaev.

It is pointed out that some of the results had been obtained by means of dynamic programming techniques by Repin (unpublished). For other applications of dynamic programming to stochastic control processes, see R. Bellman, Information and Control **1** (1958), 228–239 and *Adaptive control processes: A guided tour*, Princeton Univ. Press, Princeton, N.J., 1961 [MR **20** #3747].

R. E. Bellman (Santa Monica, Calif.)

4608:

Douce, J. L.; King, R. E. The effect of an additional non-linearity on the performance of torque-limited control systems subjected to random inputs. Proc. Inst. Elec. Engrs. C **107** (1960), 190–197.

Authors' summary: "The paper discusses a technique for improving the response of a saturating servomechanism subjected to random signals. It is shown that a non-linear error detector gives a considerable reduction in

error magnitude for a large range of input signals. It is possible to design the additional non-linearity to optimize the performance of the system for all input magnitudes. This form of non-linearity has previously been shown to effect improvement in the response of such systems to step-function and sinusoidal inputs.

"Experimental work verifies the predicted results for a particular system, and shows that considerable latitude is permitted for the characteristic of the additional non-linearity."

4609:

Fel'dbaum, A. A. Steady-state processes in the simplest discrete extremal system with random noise present. Avtomat. i Telemeh. **20** (1959), 1056–1070 (Russian. English summary); translated as Automat. Remote Control **20** (1960), 1024–1038.

Author's summary: "Investigation of the processes in discrete extremal systems with random noise present, leads to the consideration of equivalent Markoff chains. Based on this consideration, we determine the steady-state error and the optimal size of the step (the quantum size) in the simplest system. Certain generalizations of the problem posed are considered."

4610:

Sokolov, N. I. On the computation of the mean square error of processing a stationary random signal by a linear automatic control system. Avtomat. i Telemeh. **20** (1959), 1623–1634 (Russian. English summary); translated as Automat. Remote Control **20** (1960), 1577–1588.

Author's summary: "A formula is presented for the approximate computation of a convolution integral (a Duhamel integral) which has a high degree of accuracy for a large integration step. A method is given for the approximate solution (evaluation) of a double-convolution integral) which permits the rapid computation of the magnitude of the mean square error of processing a stationary random signal by an automatic control system."

4611:

Tajima, Kiyohiro. An investigation of optimum switching problems in relay or multi-position control. Mem. School Sci. Engrg. Waseda Univ. Tokyo. No. 23 (1959), 1–22.

4612:

Neĭmark, Yu. I; Šil'nikov, L. P. On the symmetric periodic motions of multistage relay systems. Avtomat. i Telemeh. **20** (1959), 1459–1466 (Russian. English summary); translated as Automat. Remote Control **20** (1960), 1422–1429.

4613:

Korolev, N. A. Approximate determination of the auto-oscillation parameters in relay systems with lagged feedback. Avtomat. i Telemeh. **20** (1959), 1467–1471 (Russian. English summary); translated as Automat. Remote Control **20** (1960), 1430–1434.

Author's summary: "An approximate method is presented for determining the parameters of the auto-oscillations of one class of relay systems on the basis of a modification of the harmonic balance method."

AUTHOR INDEX

PART B